Origami Mathematics in Education

Michael Assis
Melbourne University
School of Mathematics and Statistics

Tools and Mathematics
29 November 2016

Origami

- The Art of Folding

http://www.jccc.on.ca/assets/images/origami5.jpg

Origami

- The Art of Folding

http://img.gawkerassets.com/img/17jp3vs9qkjb6jpg/original.jpg
http://res.artnet.com/news-upload/2014/05/origami-6.jpg
http://www.joostlangeveldorigami.nl/fotos/historyoforigami/bug.jpg
http://i.ytimg.com/vi/5nZtibCqFxw/hqdefault.jpg

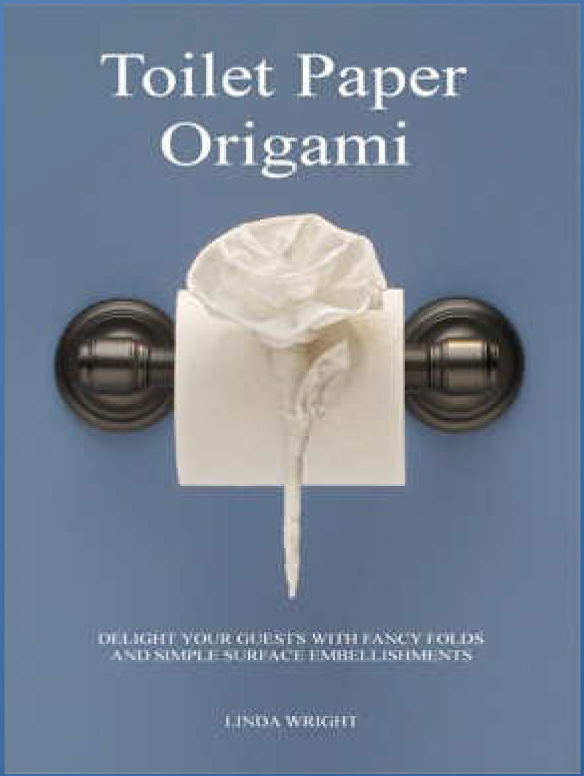
Origami

- The Art of Folding

http://illusion.scene360.com/wp-content/themes/sahara-10/submissions/2012/10/jun_mitani_03.jpg
http://farm5.static.flickr.com/4107/4946857347_a17b3e7900.jpg
http://giangdinh.com/wp-content/uploads/2013/09/prayer.jpg

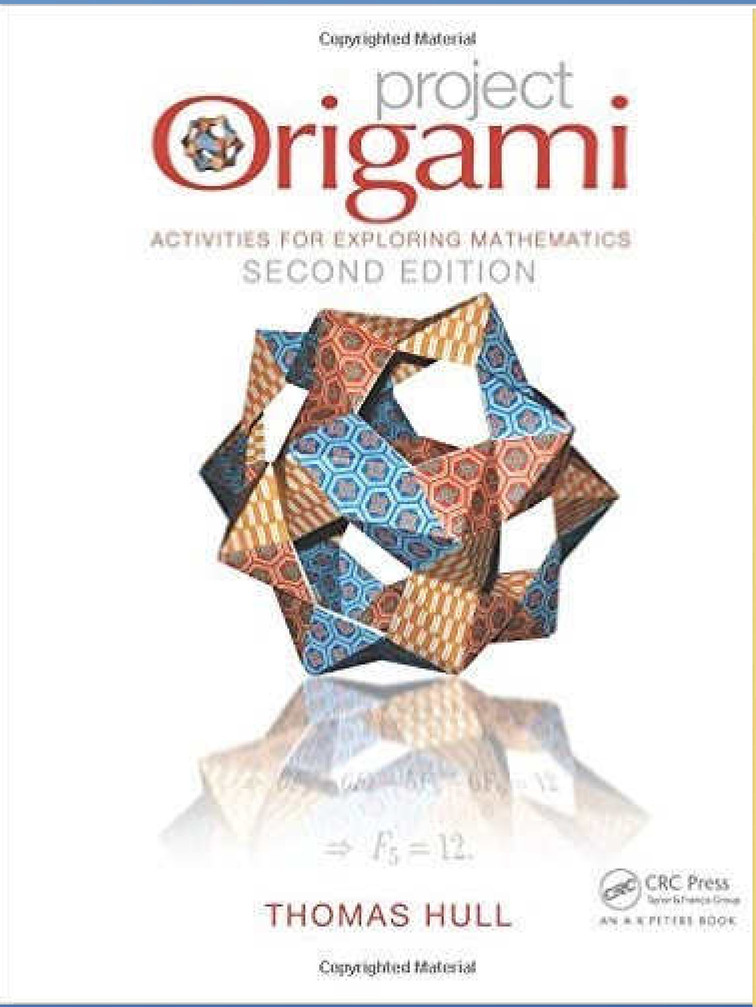
Origami

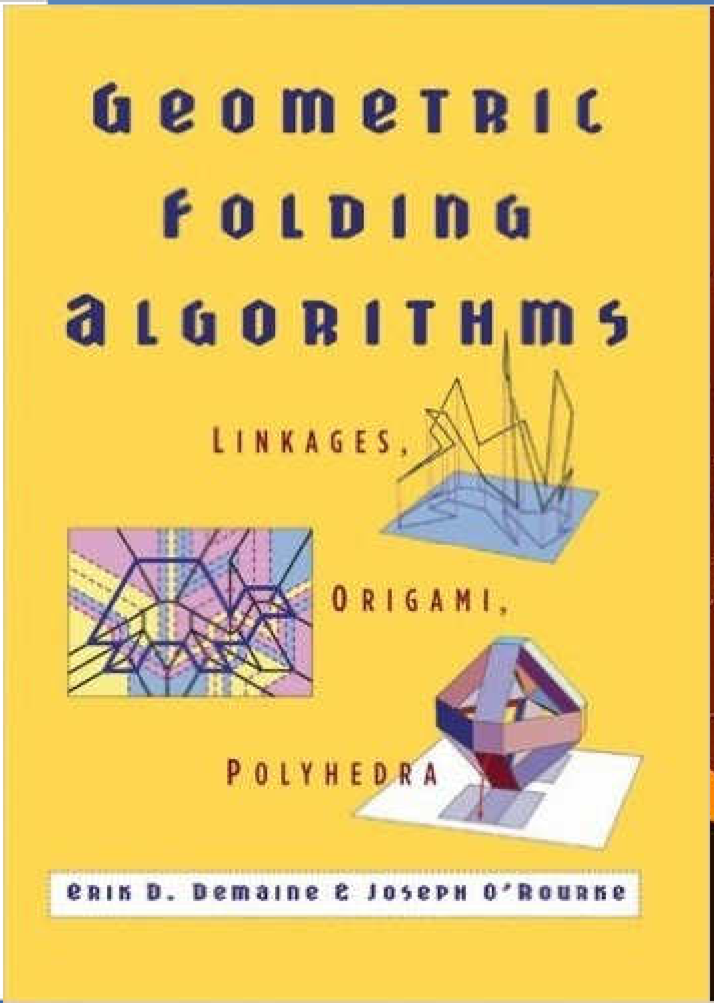
- The Art of Folding

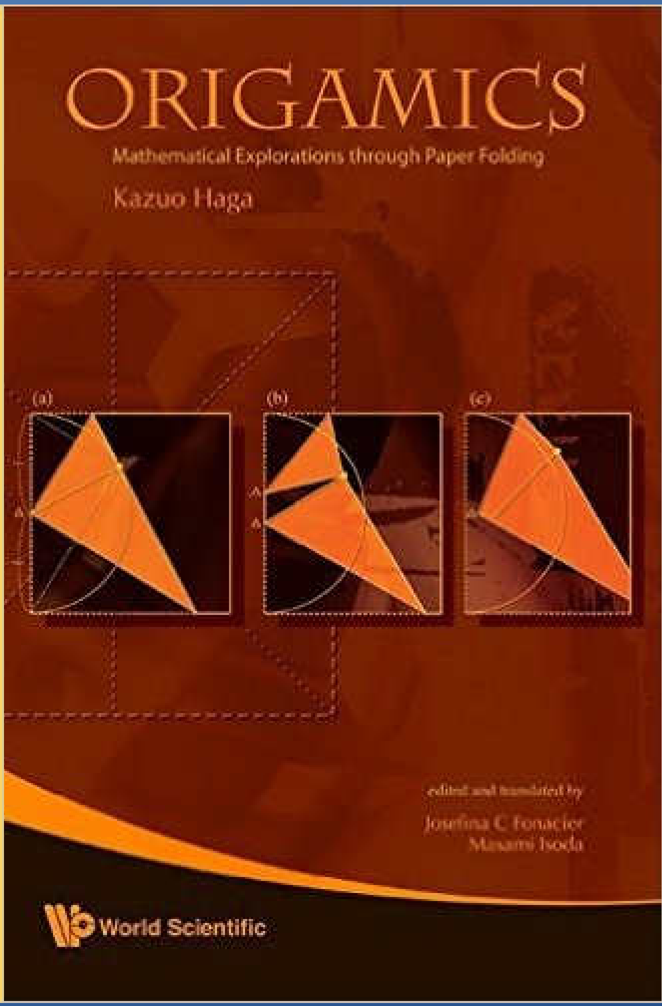


https://c2.staticflickr.com/4/3530/5835802683_a7ca138ff9.jpg
http://www.tporigami.com/wp-content/uploads/2010/09/ToiletPaperOrigami_Cover.jpg
https://nrgtucker.files.wordpress.com/2012/12/20121223-183459.jpg
http://strictlypaper.com/blog/wp-content/uploads/2013/03/nintai-origami-inspired-dresses-strictlypaper-1.jpg

Origami in the Classroom







Origami Resources

1D Origami

Folding In Half

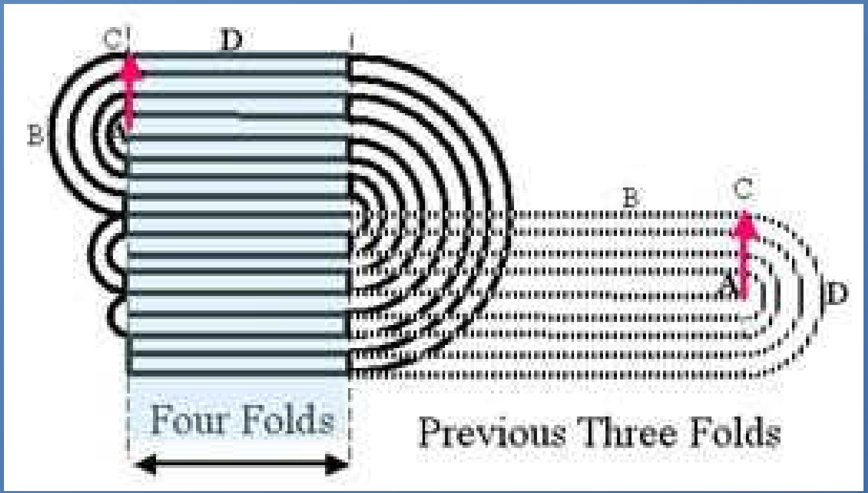
- How many times can you fold paper in half?
  - 8 times?

Folding In Half

- How many times can you fold paper in half?
  - 8 times?
- Is there an upper limit?

Folding In Half

- Britney Gallivan 2001



$$L = \frac{\pi \cdot t}{6} \cdot (2^n + 4)(2^n - 1)$$

$$W = \pi \, t \, 2^{3(n-1)/2}$$

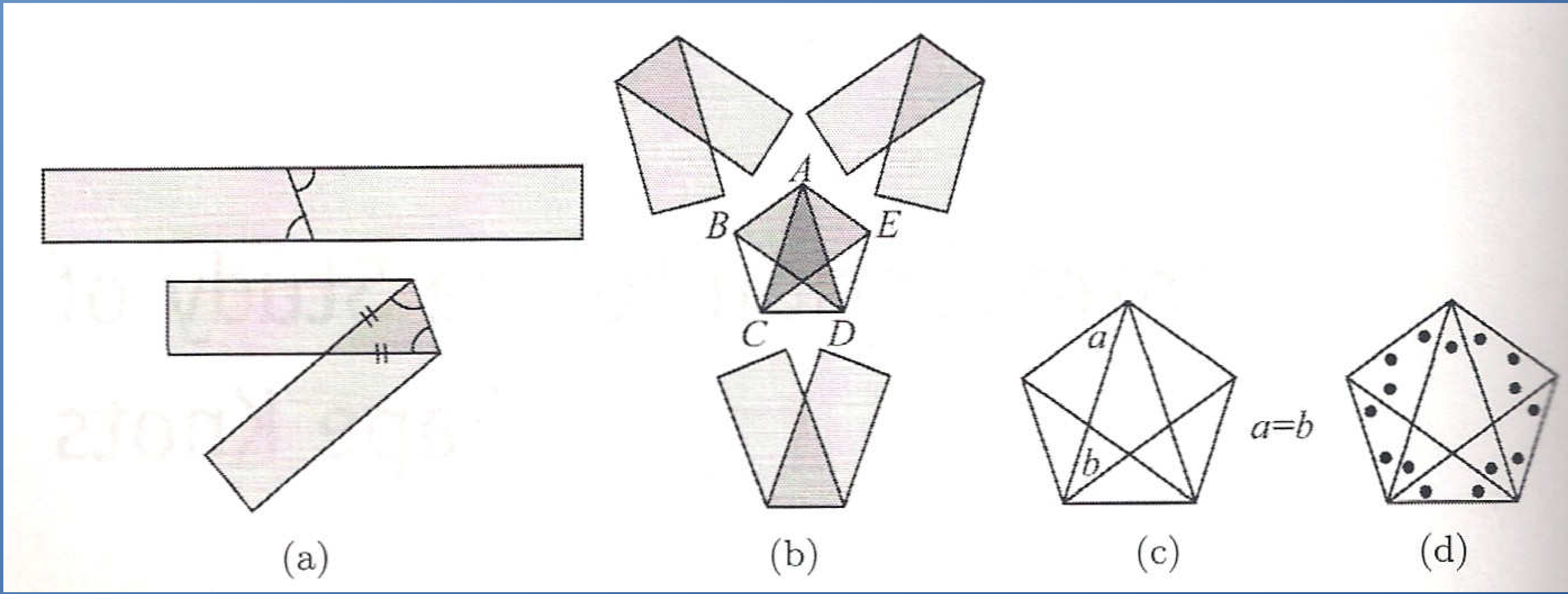
Activity 1

Parabolas

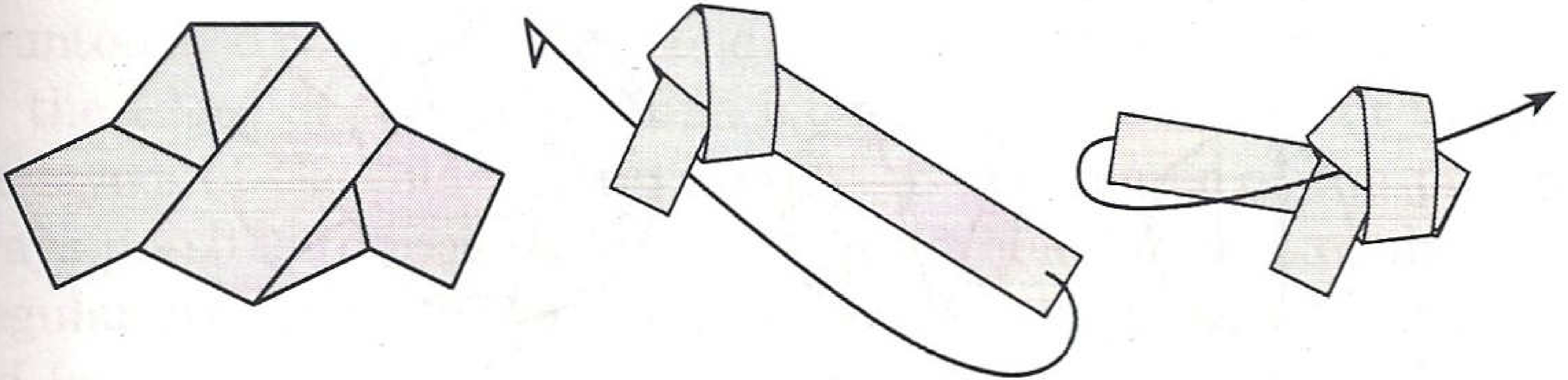
- Why does it work?
- Can other conics be constructed?
- What if you use non-flat paper?
- What can we learn concerning:
  - Parabolas ?
  - Envelopes?
  - Derivatives?
  - Tangents?
  - Convergence of sequences?

Knots

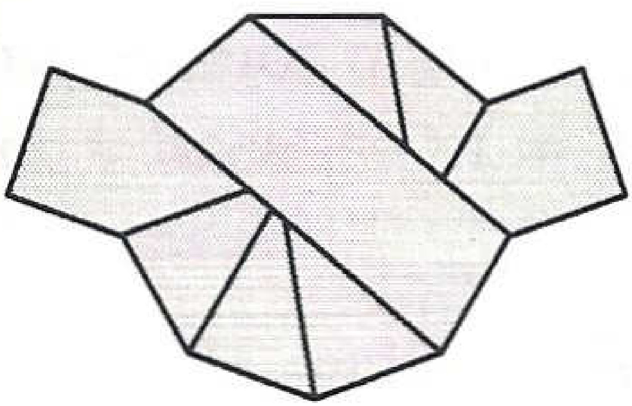
Knots



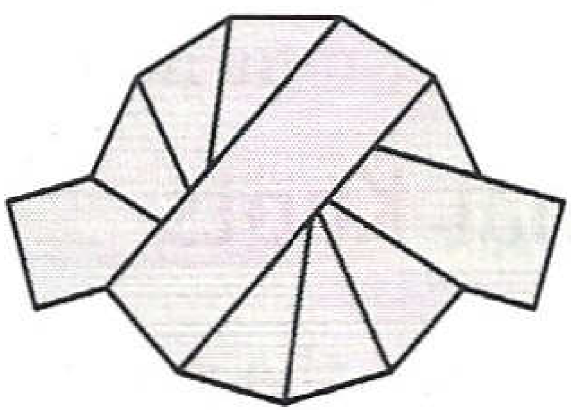
Knots



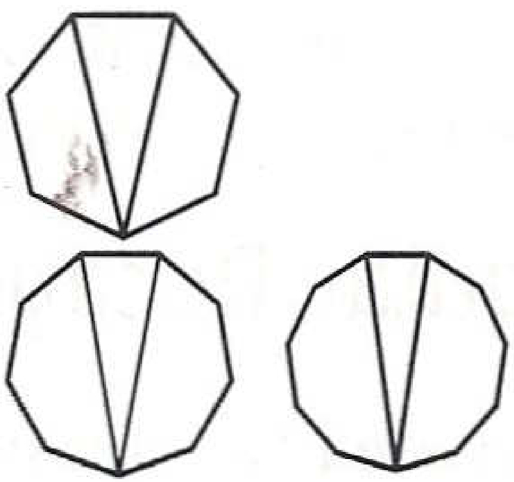
(a)



(b)

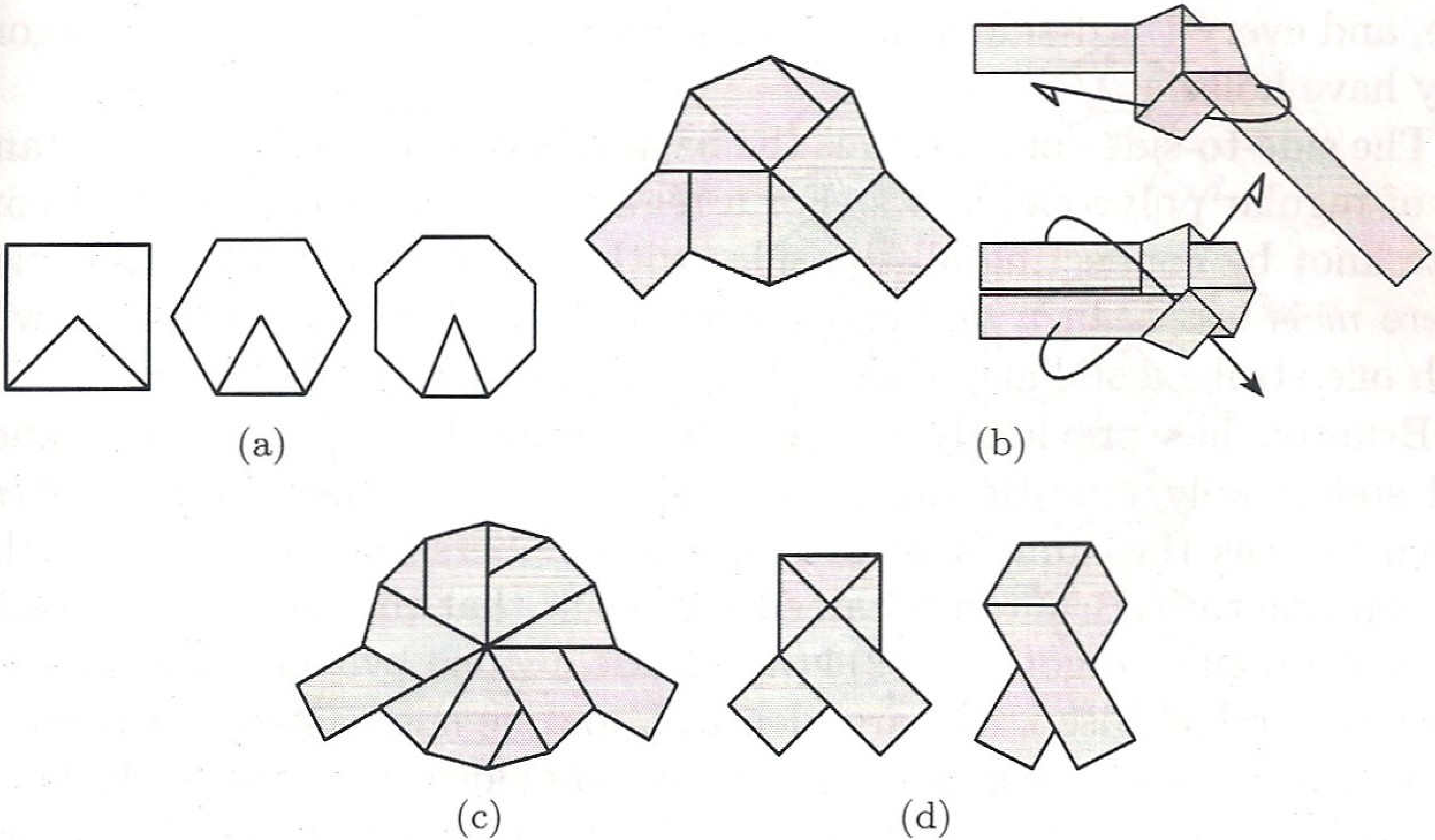


(c)

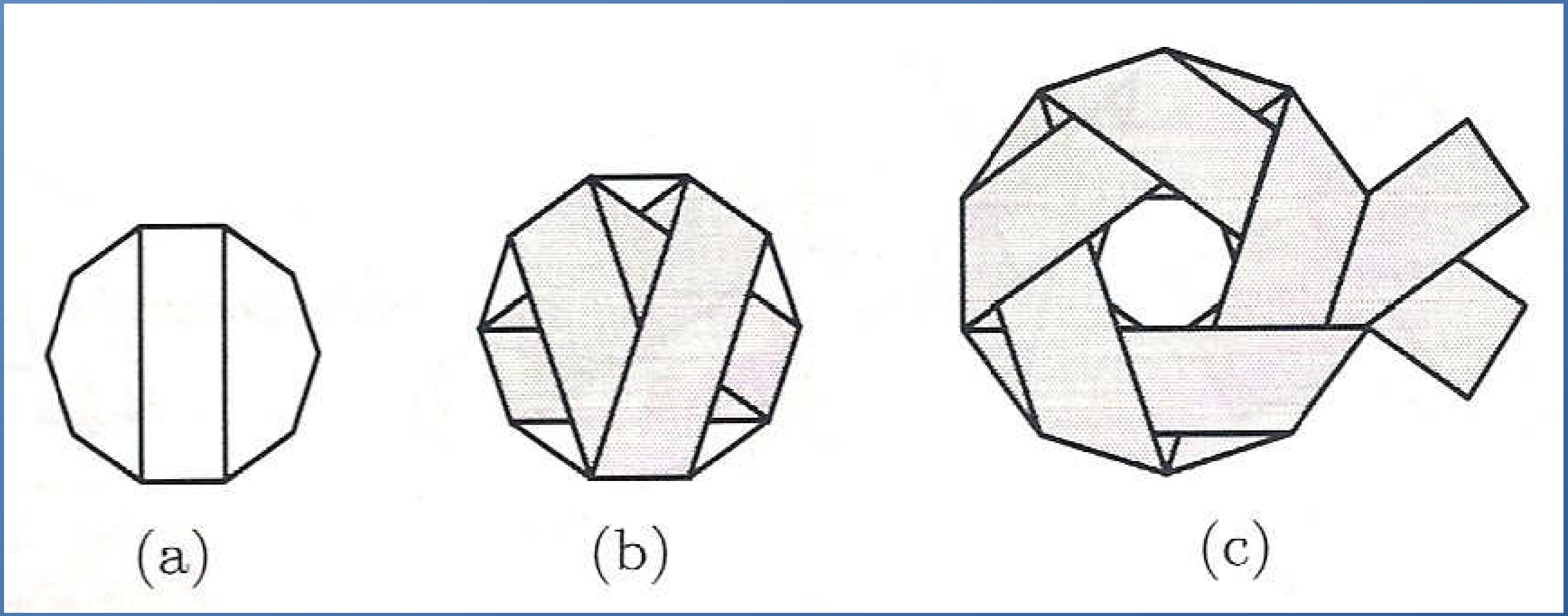


(d)

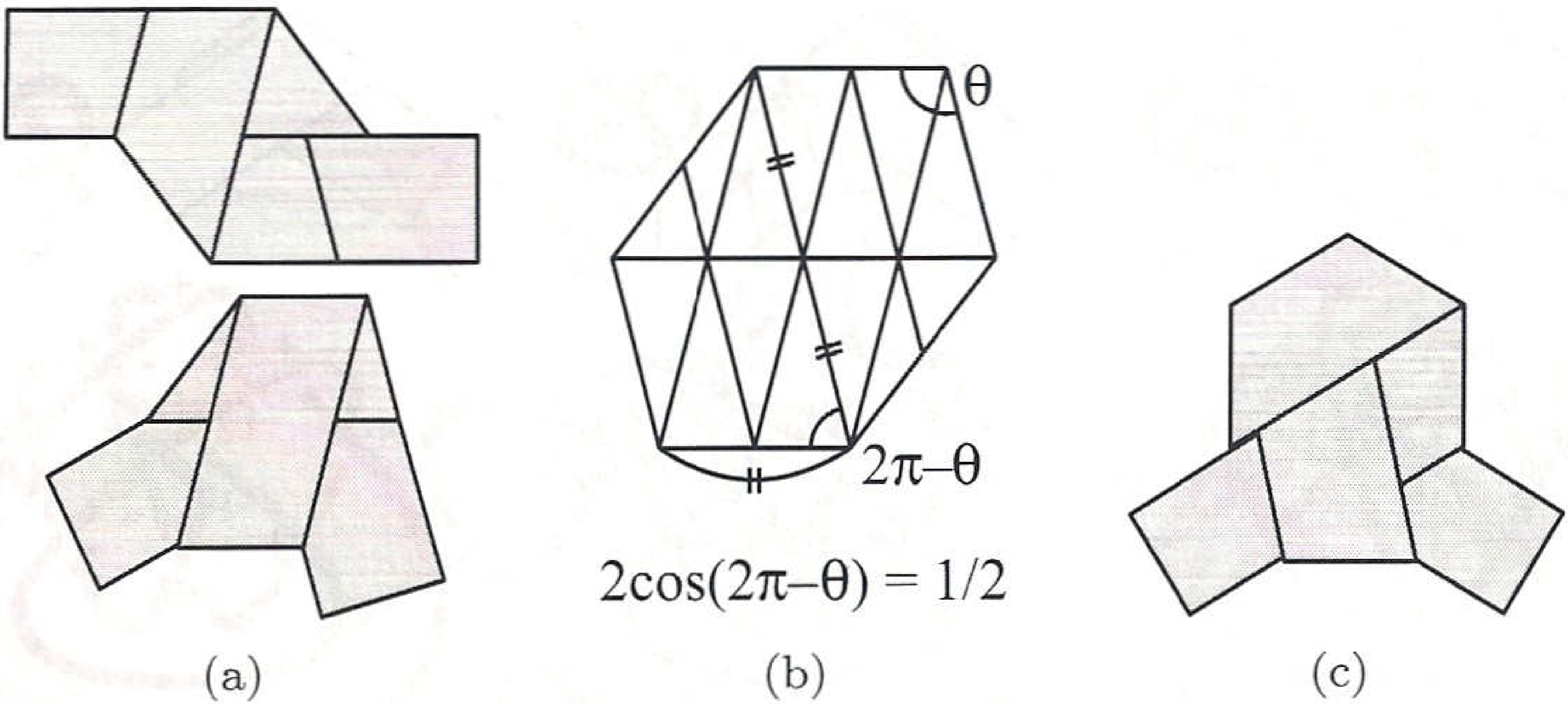
Knots



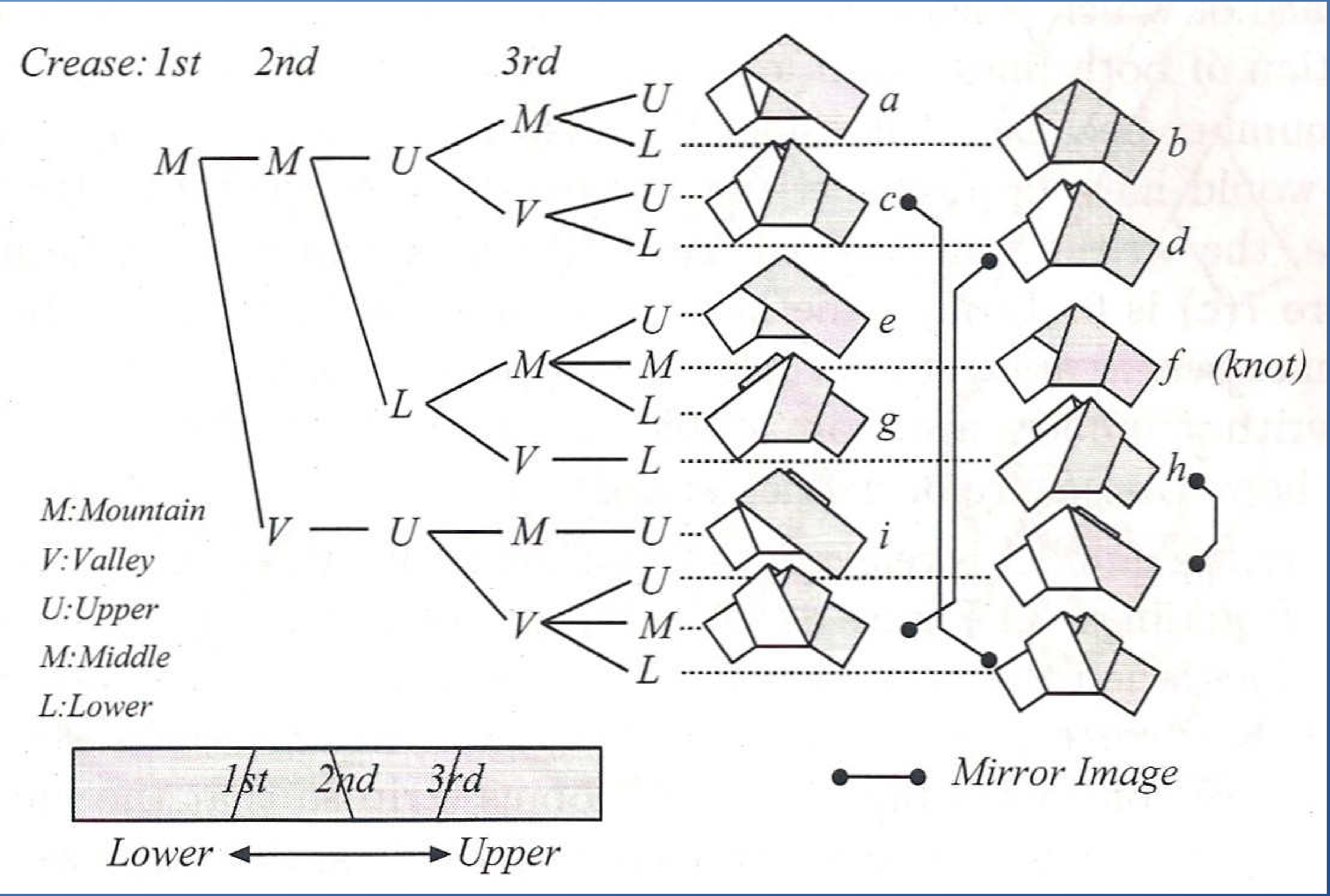
Knots



Knots

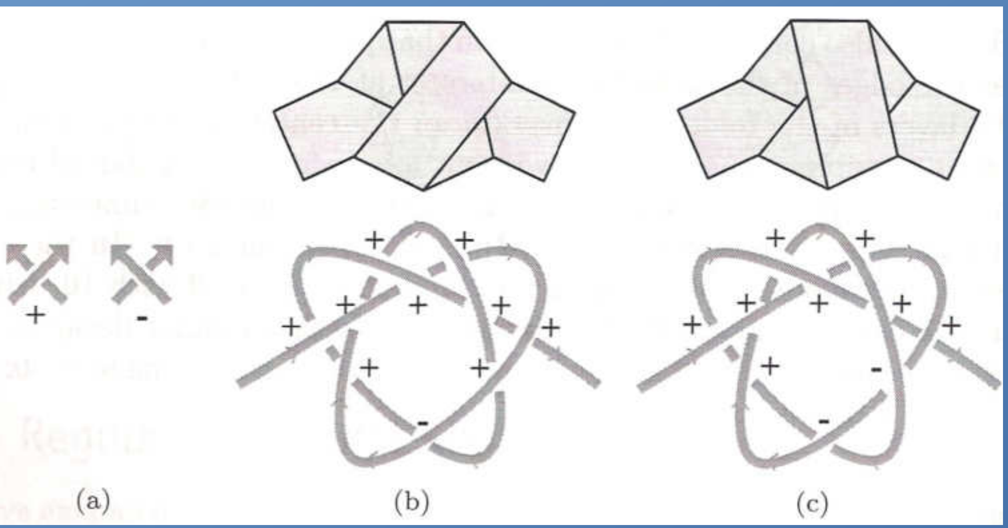


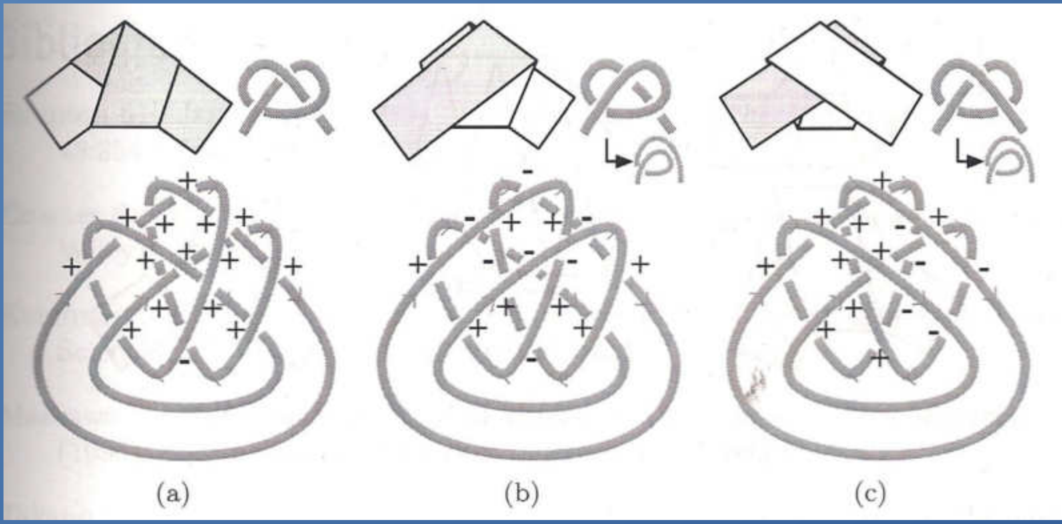
Knots

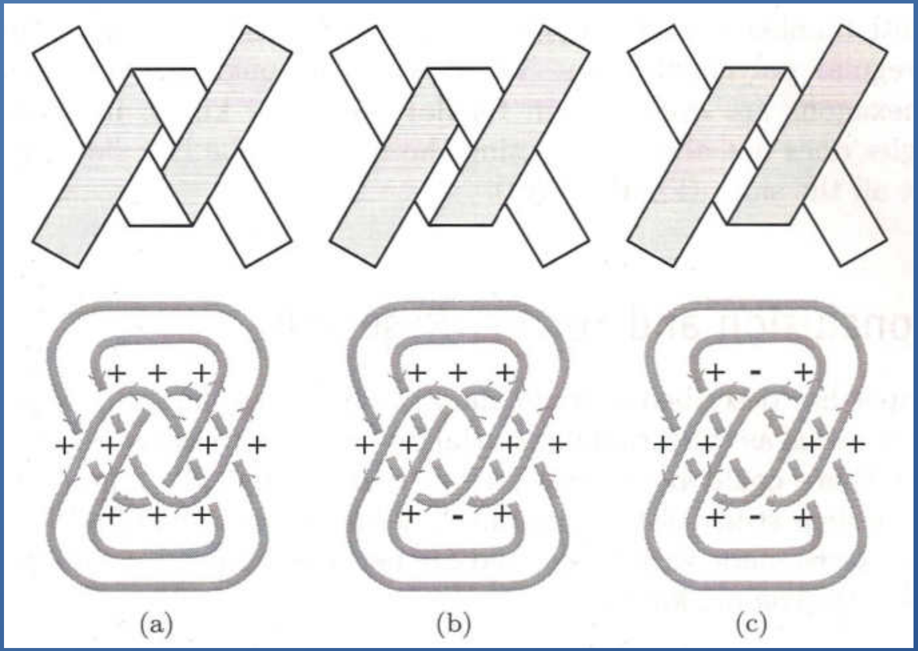


Crease: 1st 2nd 3rd
M M U M U
L
V U
L
L M U
M
L
V L
V U M U
V U
M
L
a
b
c
d
e
f (knot)
g
h
i
M:Mountain
V:Valley
U:Upper
M:Middle
L:Lower
1st 2nd 3rd
Lower Upper
Mirror Image

Knots







Knots

- Explorations:
  - Perimeter, area
  - Irregular patterns
  - Enumerations
  - Knot theory, topology

Activity 2

- Fujimoto approximation

Fujimoto Approximation

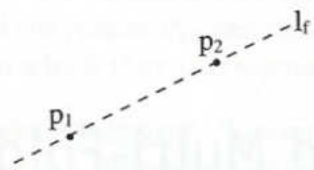
- Error is halved at each operation
- Repeating left-right pattern represented as the binary expansion of 1/n
  - 1/5: .00110011...
  - 1/7: .011011011...

Between 1D and 2D

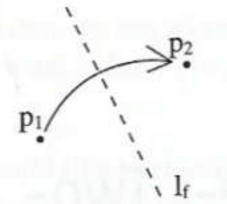
Origami Constructions

- What geometric constructions are possible?

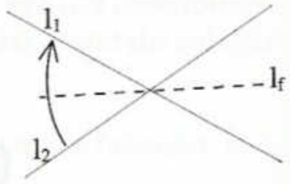
Origami Constructions



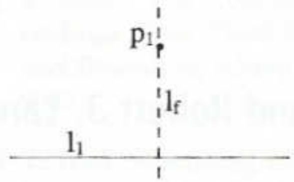
(O1) Given two points p_1 and p_2, we can fold a line connecting them.



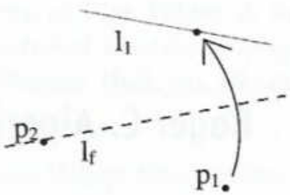
(O2) Given two points p_1 and p_2, we can fold p_1 onto p_2.



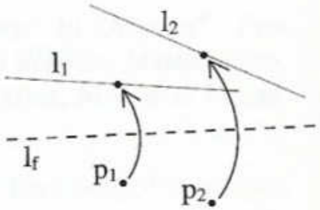
(O3) Given two lines l_1 and l_2, we can fold line l_1 onto l_2.



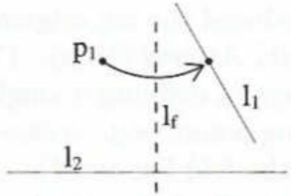
(O4) Given a point p_1 and a line l_1, we can make a fold perpendicular to l_1 passing through the point p_1.



(O5) Given two points p_1 and p_2 and a line l_1, we can make a fold that places p_1 onto l_1 and passes through the point p_2.

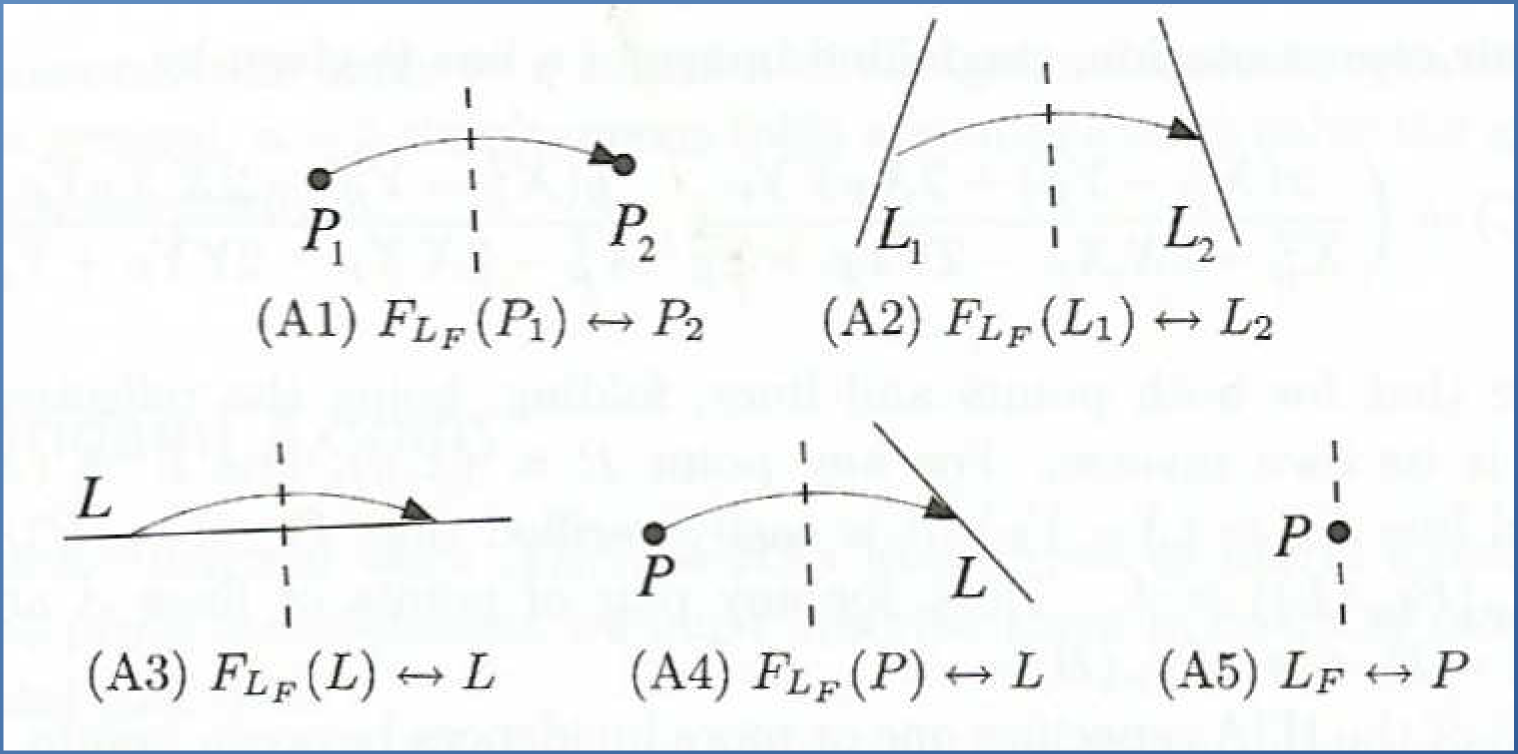


(O6) Given two points p_1 and p_2 and two lines l_1 and l_2, we can make a fold that places p_1 onto line l_1 and places p_2 onto line l_2.



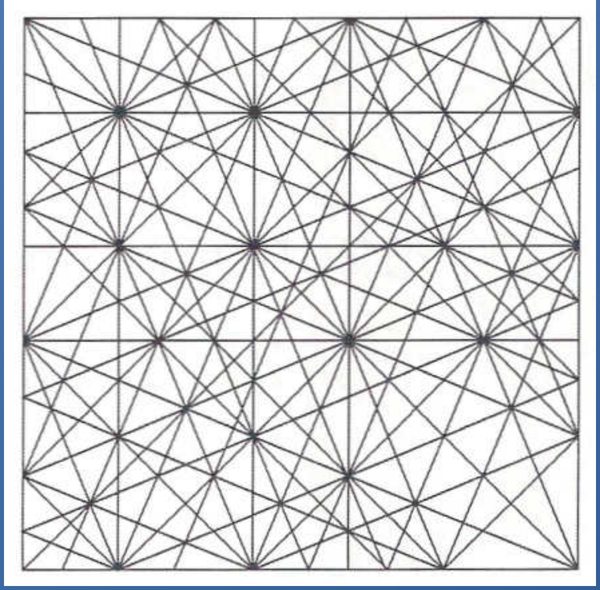
(O7) Given a point p_1 and two lines l_1 and l_2, we can make a fold perpendicular to l_2 that places p_1 onto line l_1.

Origami Constructions



Origami Constructions

- 22.5 degree angle restriction
  - All coordinates of the form $\frac{m+n\sqrt{2}}{2^l}$ are constructible
  - Algorithm linear in l, log(m), log(n)



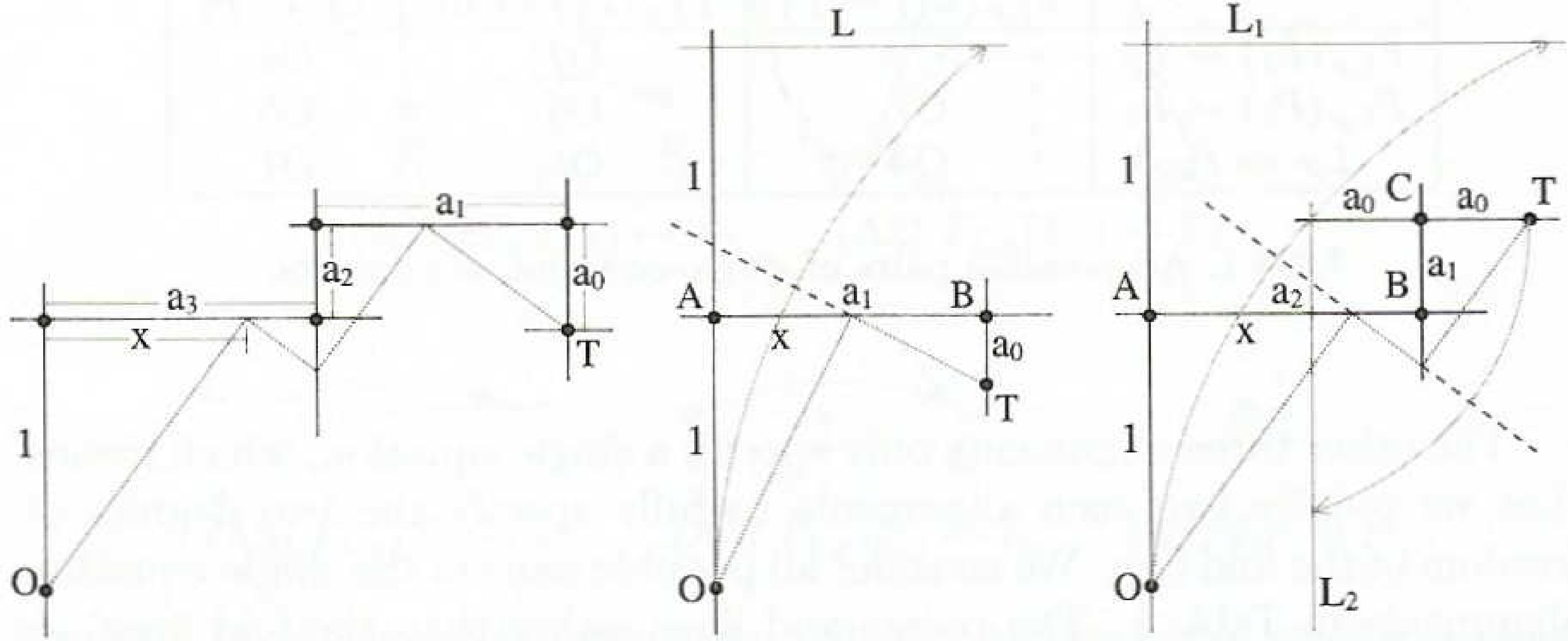
Origami Constructions

- More generally:
  - Constructible numbers of the form $2^m 3^n$
  - Angle trisection, cube doubling possible
  - Roots of the general cubic

Origami Constructions

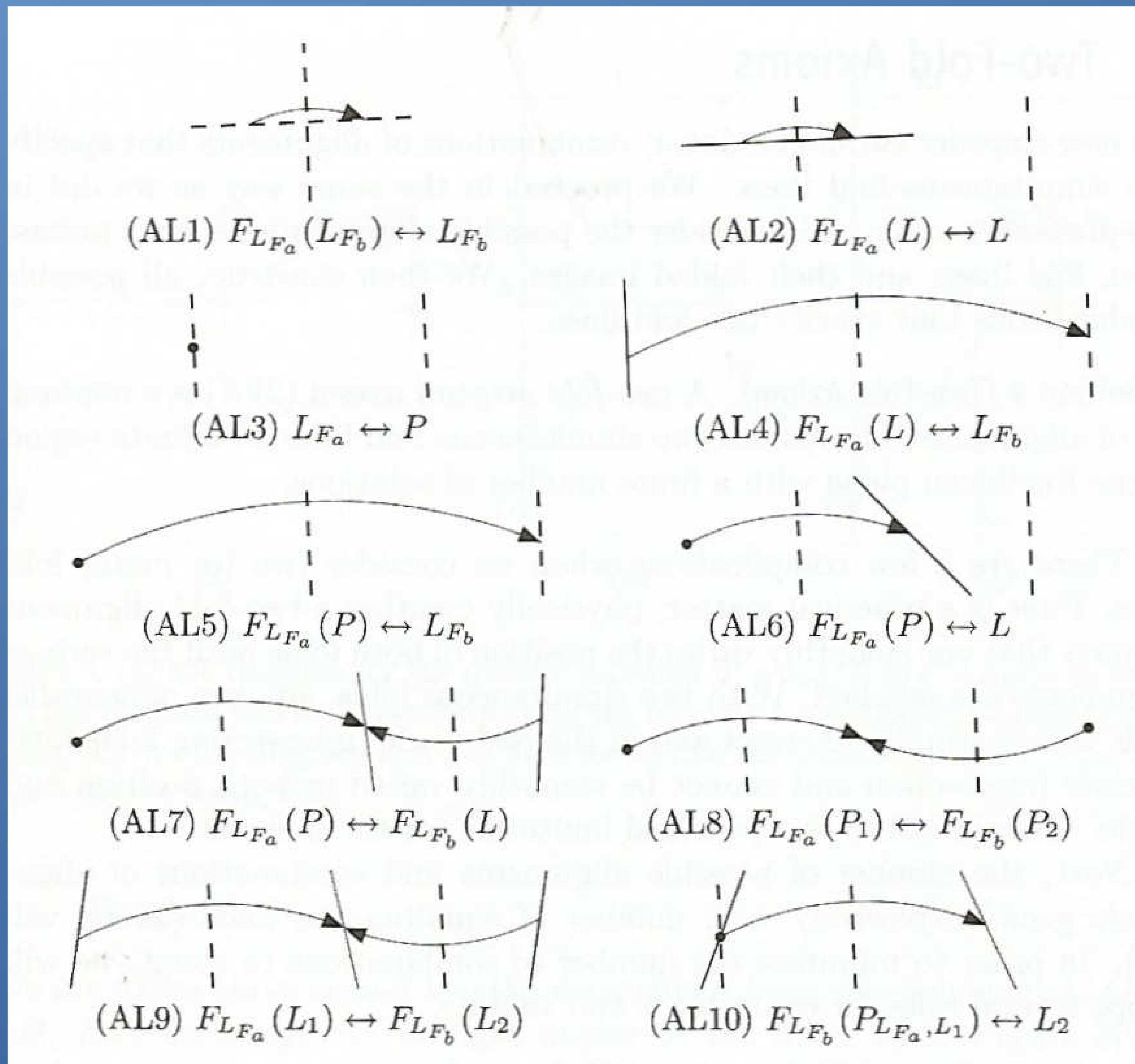
- Polynomial root finding, Lill's method

$$x^4 - a_3 x^3 + a_2 x^2 - a_1 x - a_0 = 0 \qquad x^2 - a_1 x - a_0 = 0 \qquad x^3 - a_2 x^2 + a_1 x - a_0 = 0$$



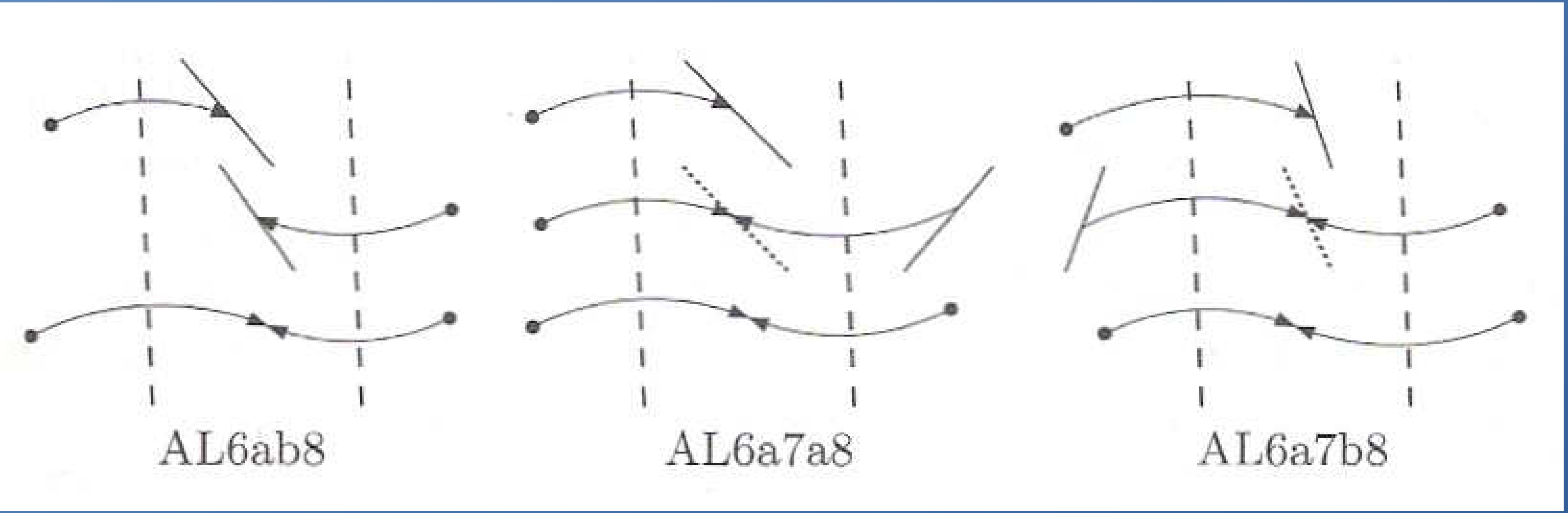
(a) (b) (c)

Origami Constructions



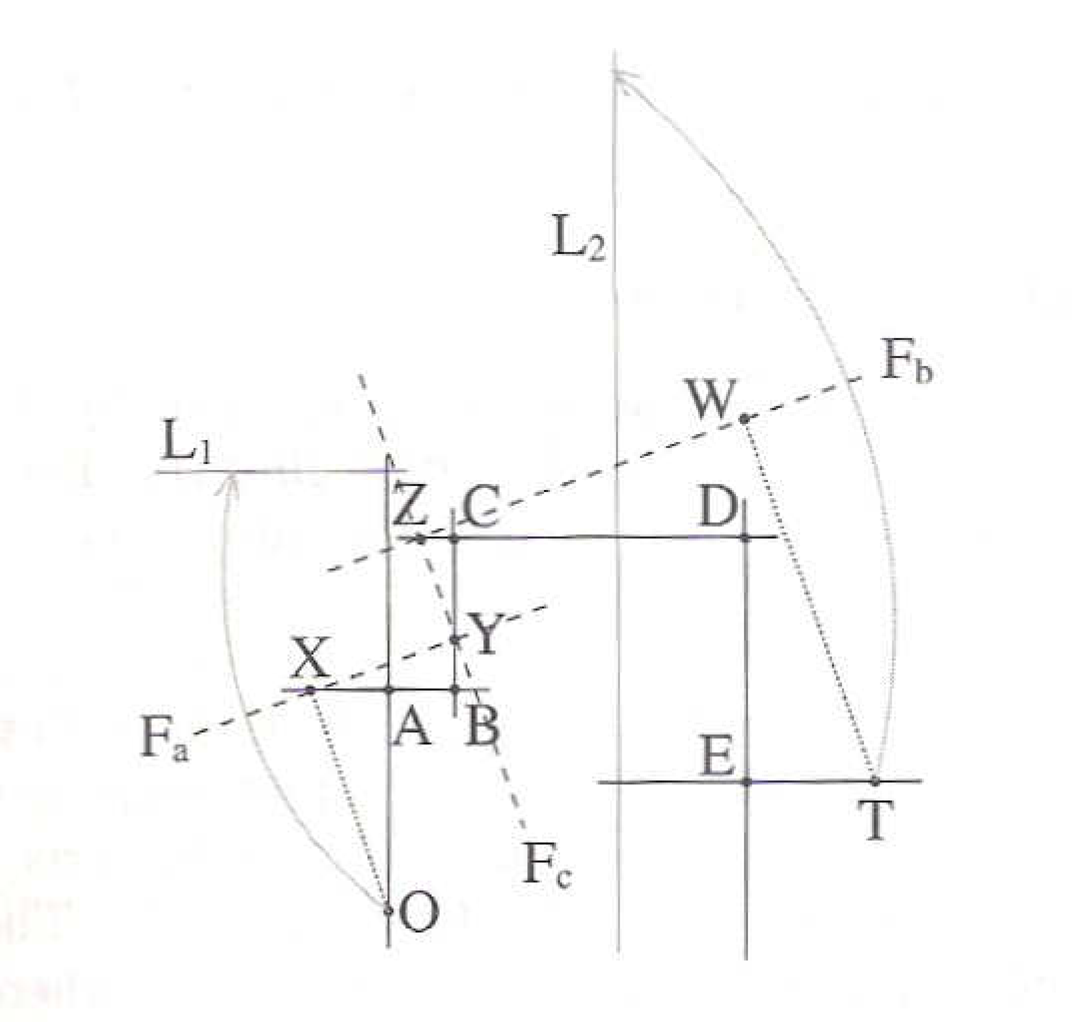
Origami Constructions

- 489 distinct two-fold line constructions



Origami Constructions

- General quintic construction



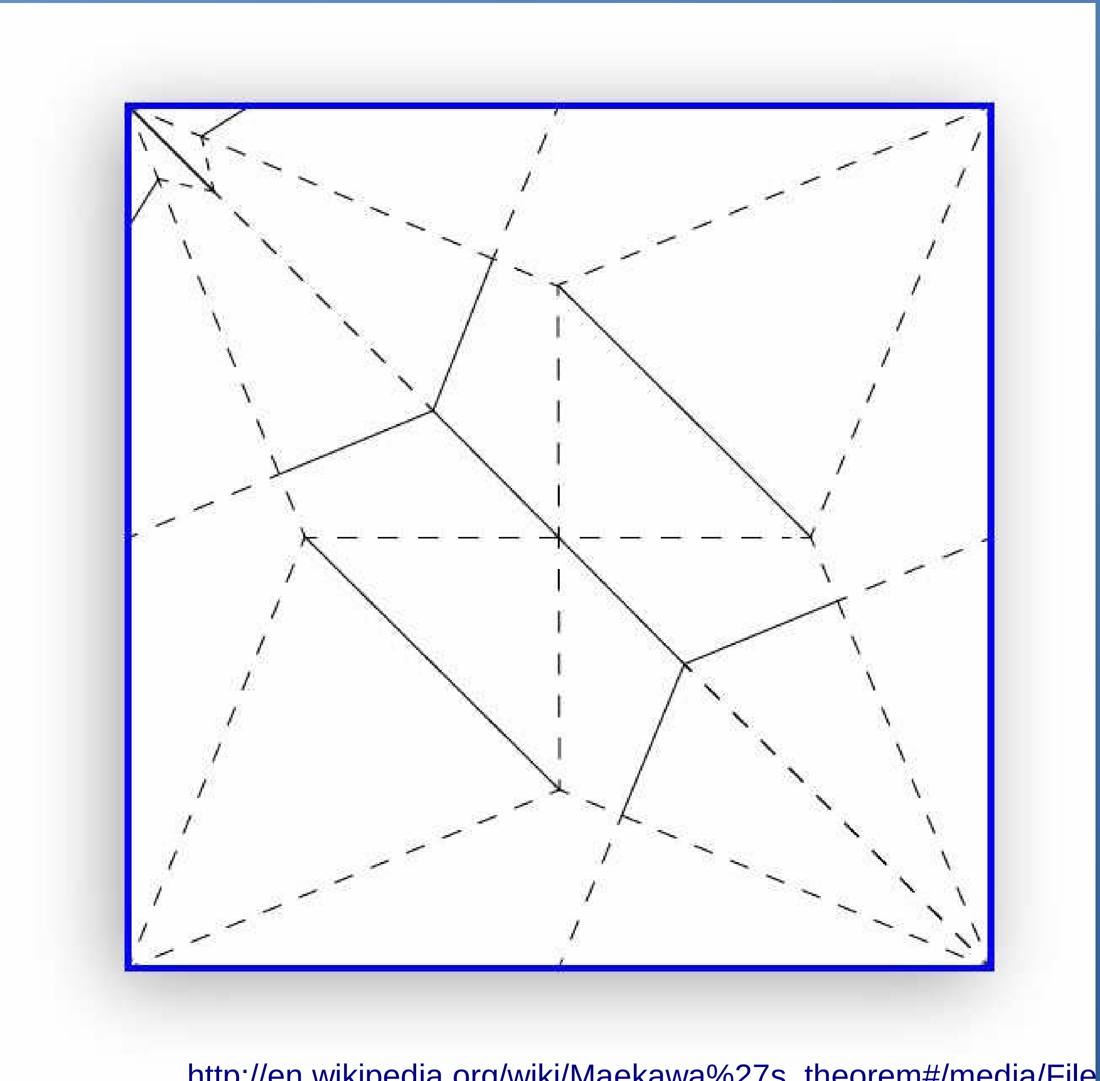
Origami Constructions

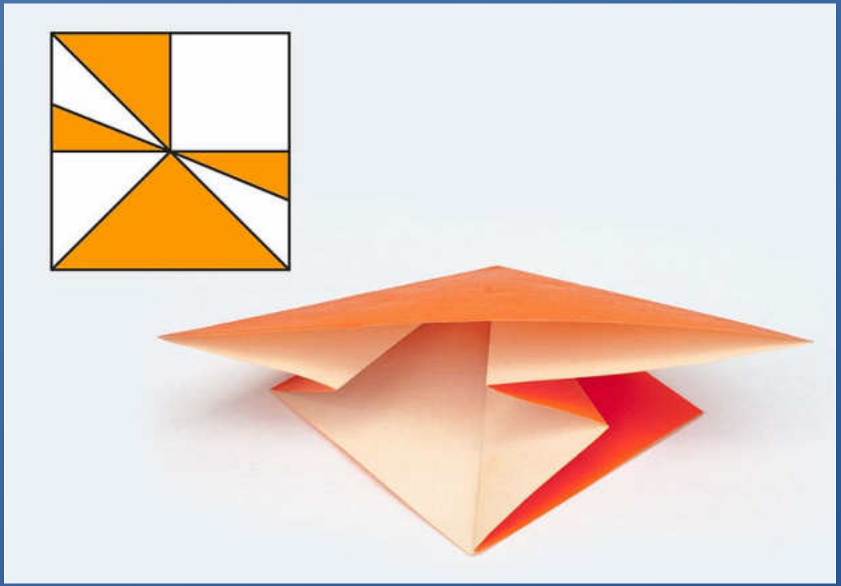
- Higher order equations, real solutions
  - Order *n* requires (*n*-2) simultaneous folds
- What can we learn concerning:
  - Polynomial roots
  - Geometric constructions
  - Field theory
  - Galois theory

2D Folding

Flat Foldability Theorems

- Maekawa's theorem: |M-V|=2, even degrees

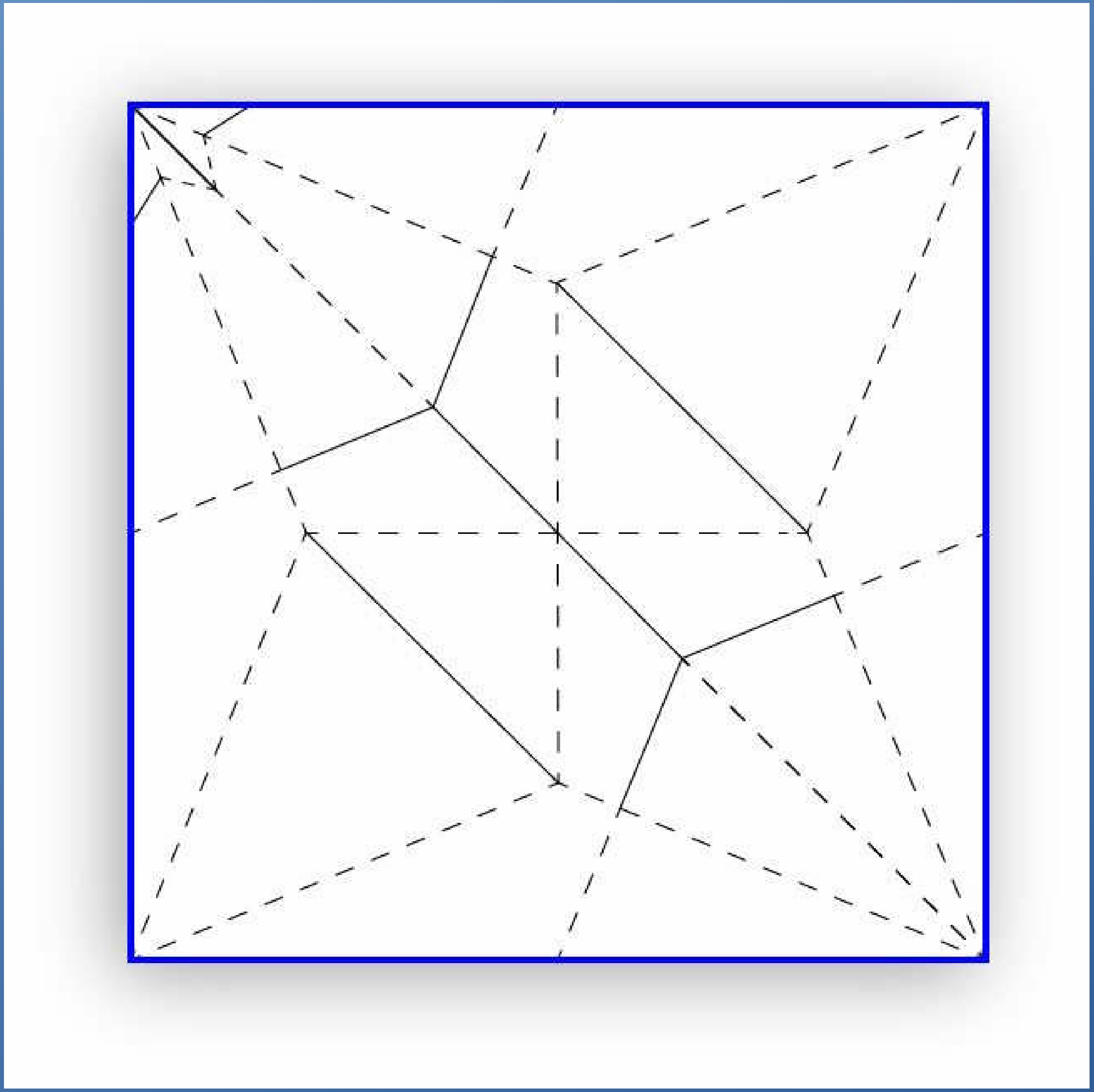


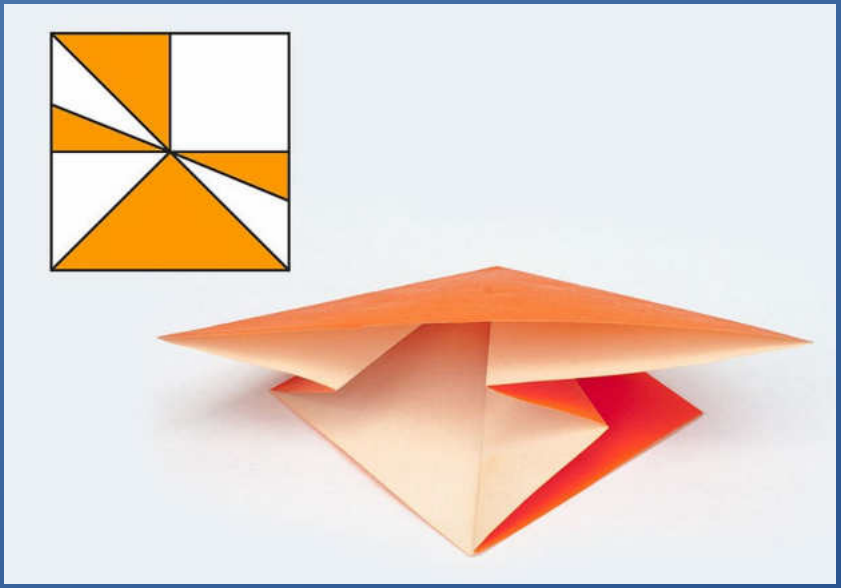


http://en.wikipedia.org/wiki/Maekawa%27s_theorem#/media/File:Kawasaki%27s_theorem.jpg

Flat Foldability Theorems

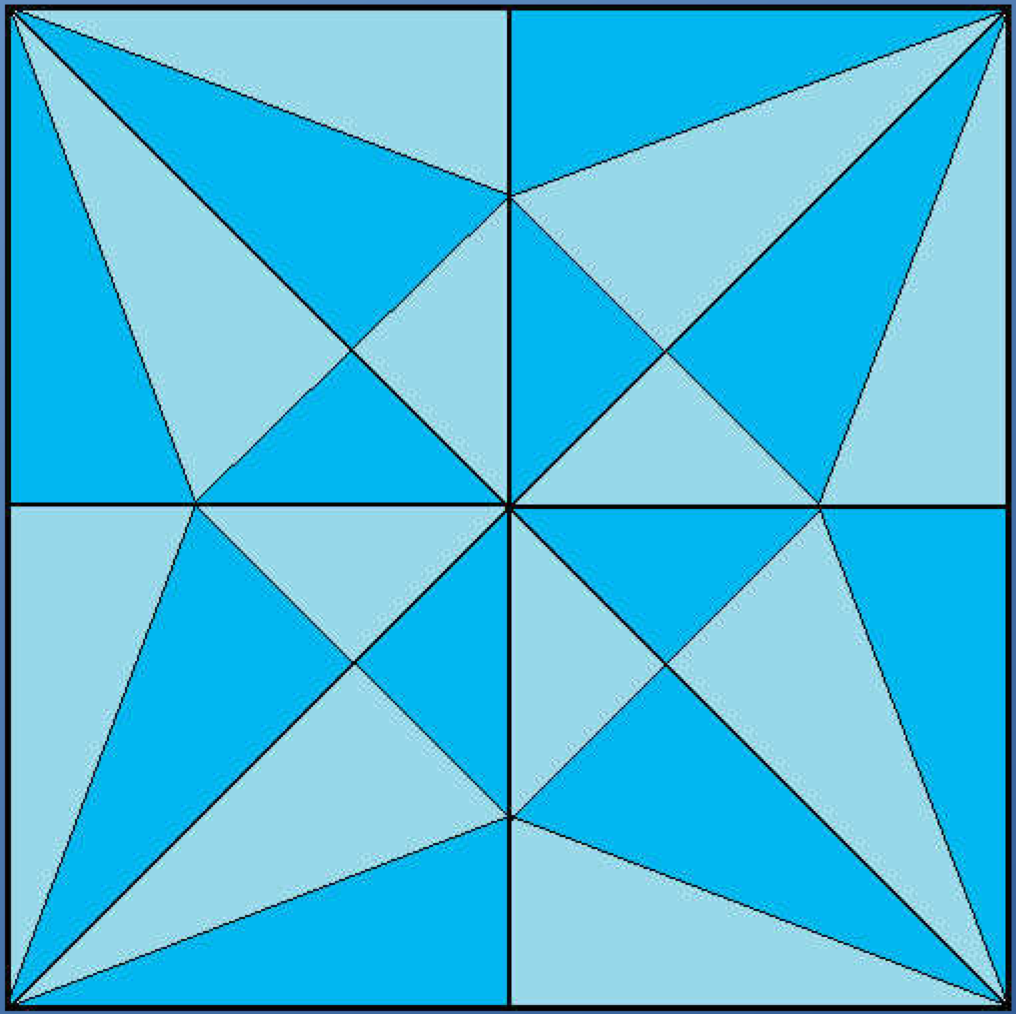
- Kawasaki's theorem: sum of alternating angles equals 180°

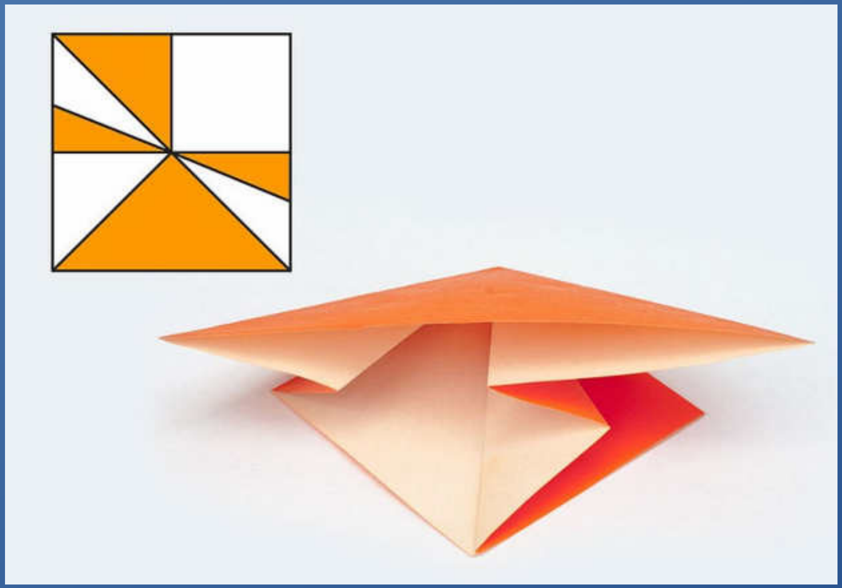




Flat Foldability Theorems

- Crease patterns are two-colorable

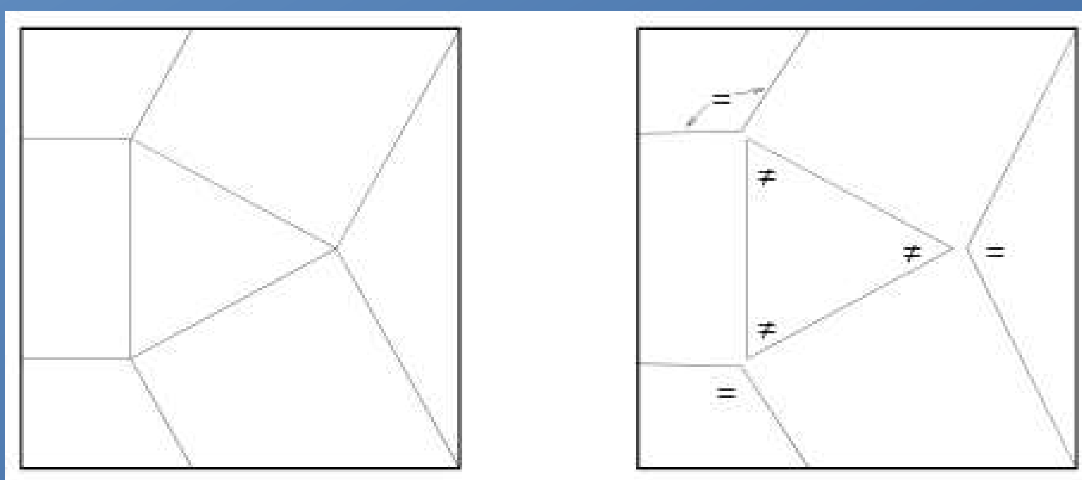


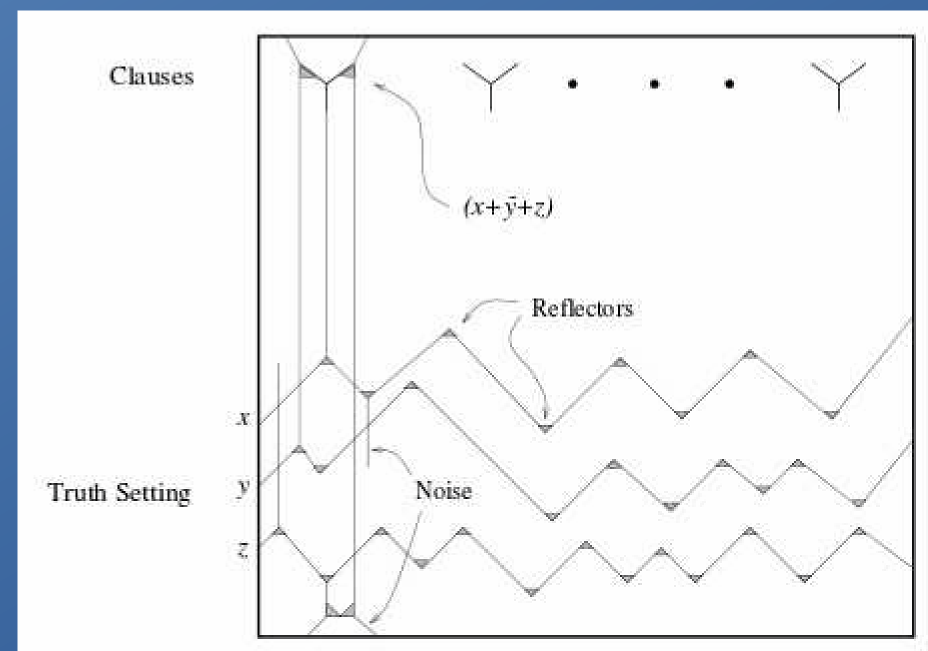


http://en.wikipedia.org/wiki/Mathematics_of_paper_folding#/media/File:Lang_rule_one.png

Flat Foldability is Hard

- Deciding flat-foldability is NP-complete





The Complexity of Flat Origami. Marshall Bern , Barry Hayes. Proceedings of the 7th Annual ACM-SIAM Symposium on Discrete Algorithms, 1996.

Circle Packing

- What is a flap?

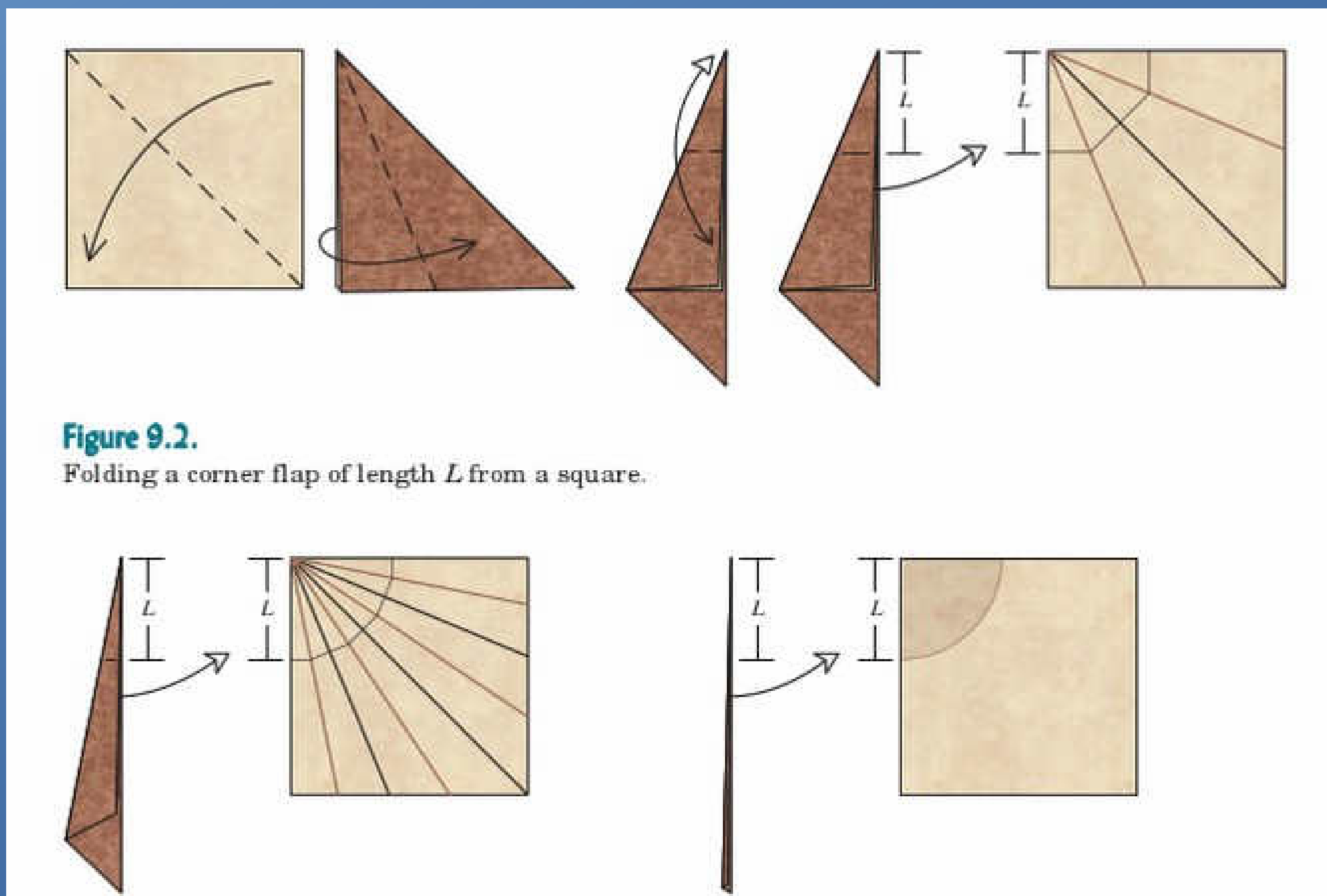
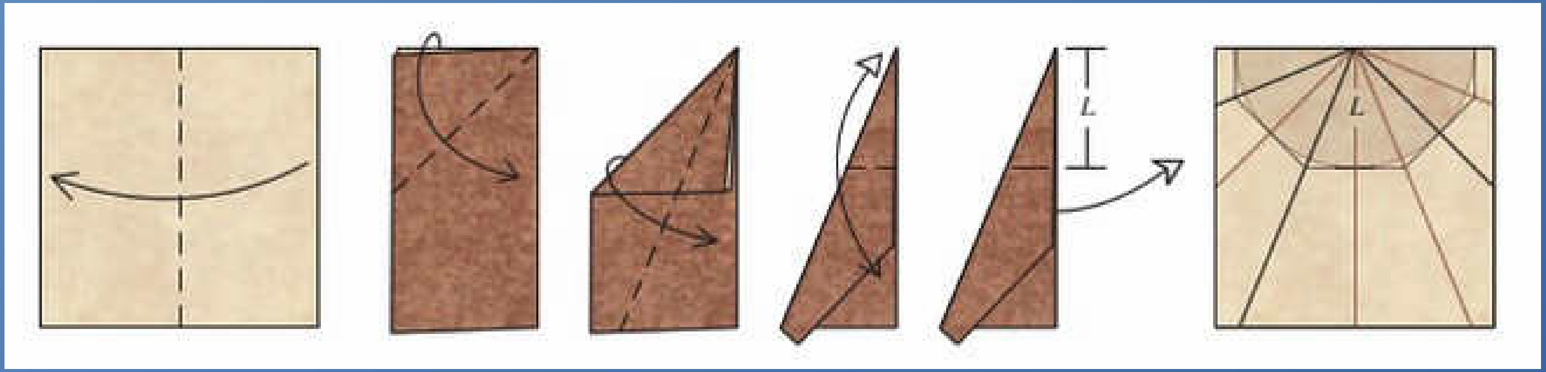


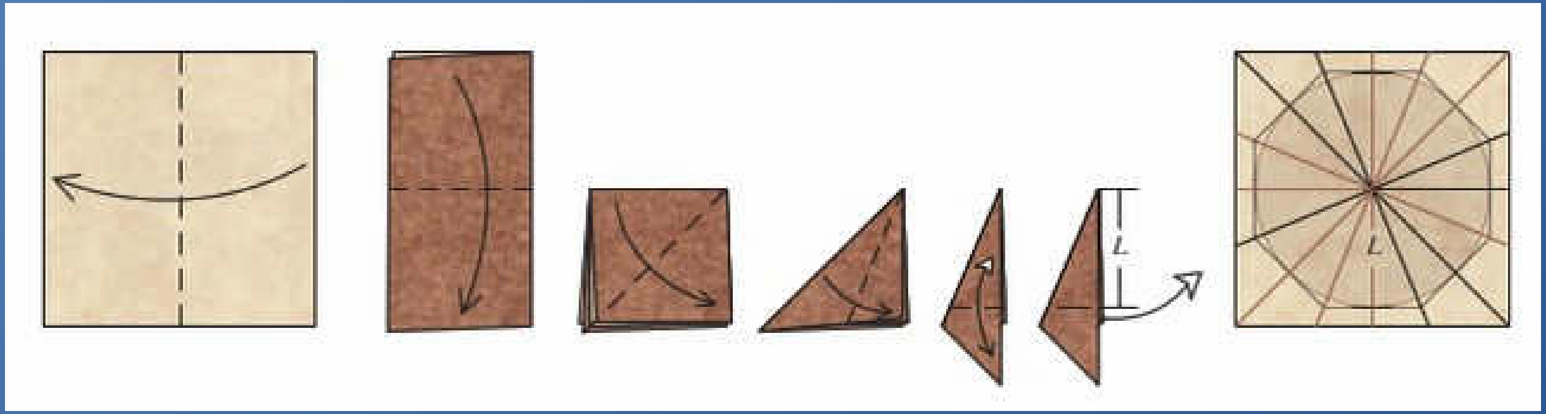
Figure 9.2.
Folding a corner flap of length L from a square.

Origami Design Secrets, by Robert Lang

Circle Packing

- What is a flap?

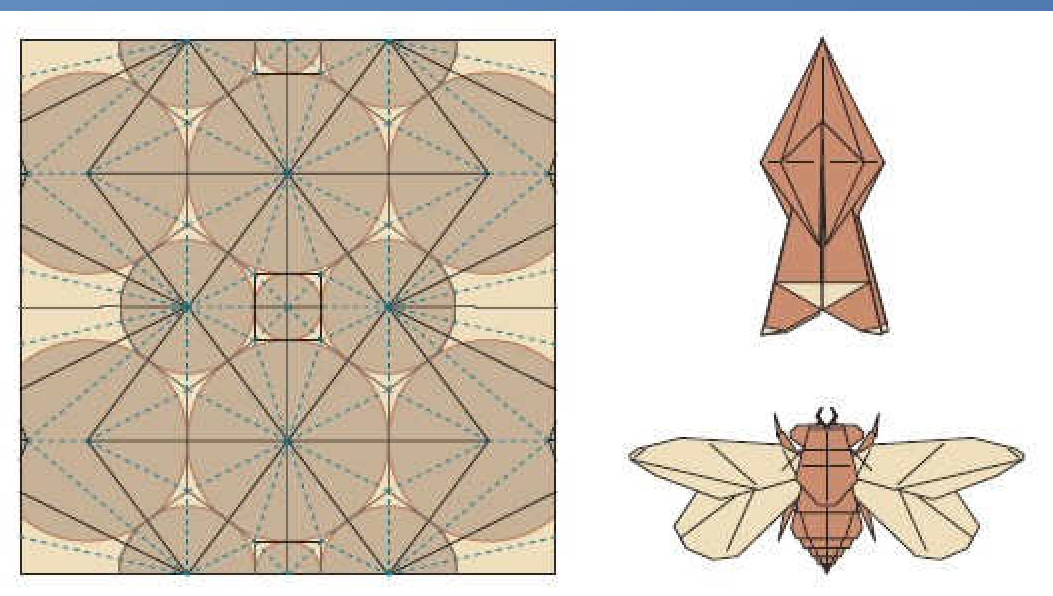


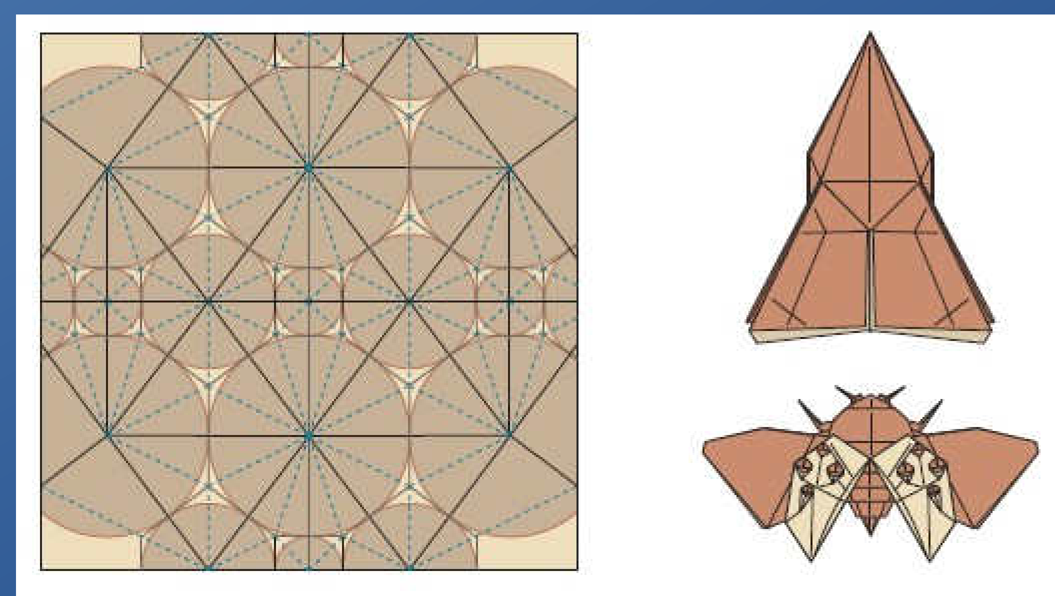


Origami Design Secrets, by Robert Lang

Circle Packing

- Understanding crease patterns using circles

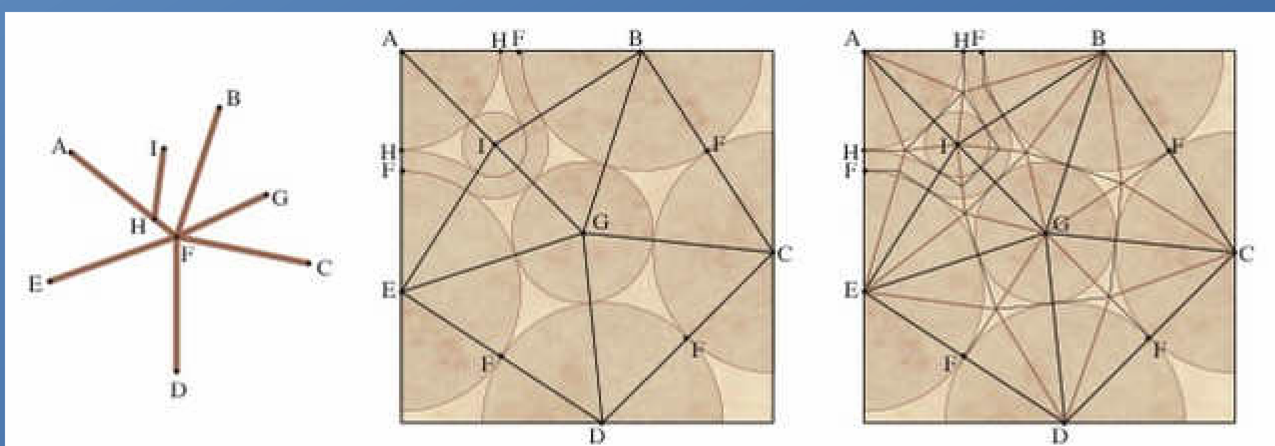




Origami Design Secrets, by Robert Lang

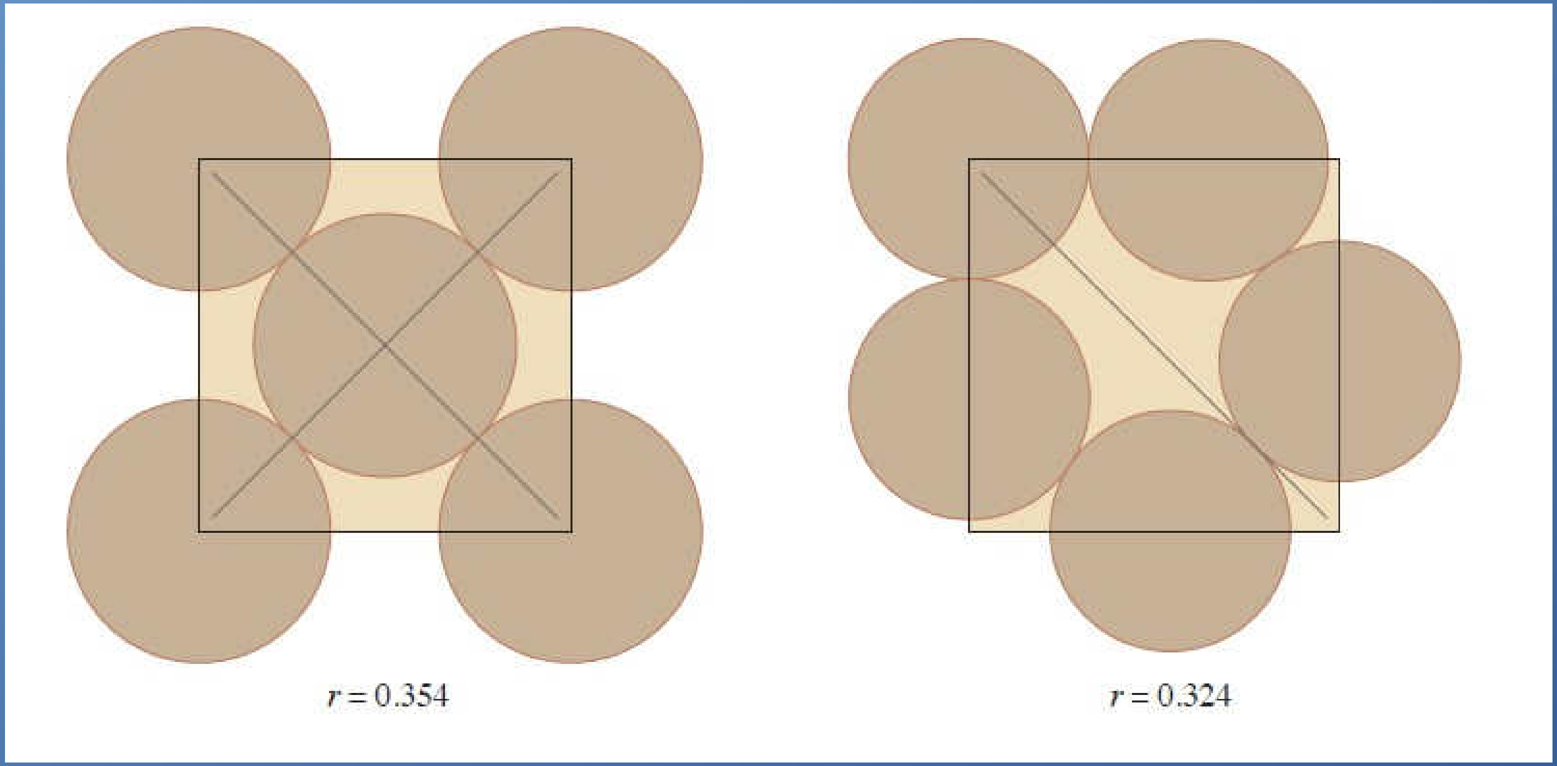
Circle Packing

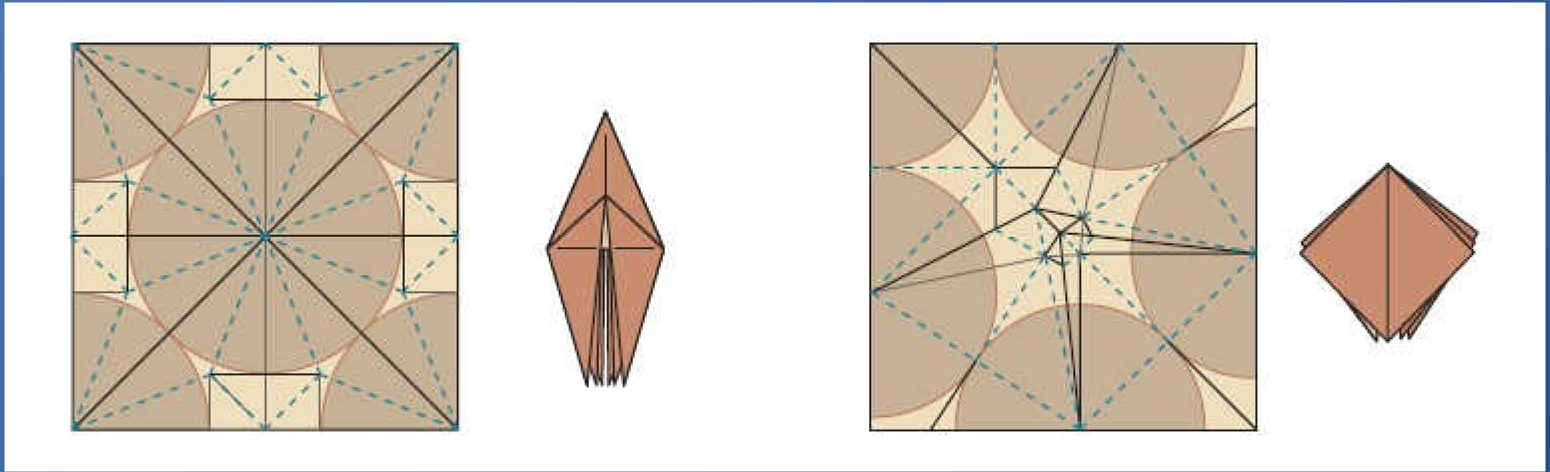
- Design algorithm
  - Uniaxial tree theory
  - Universal molecule



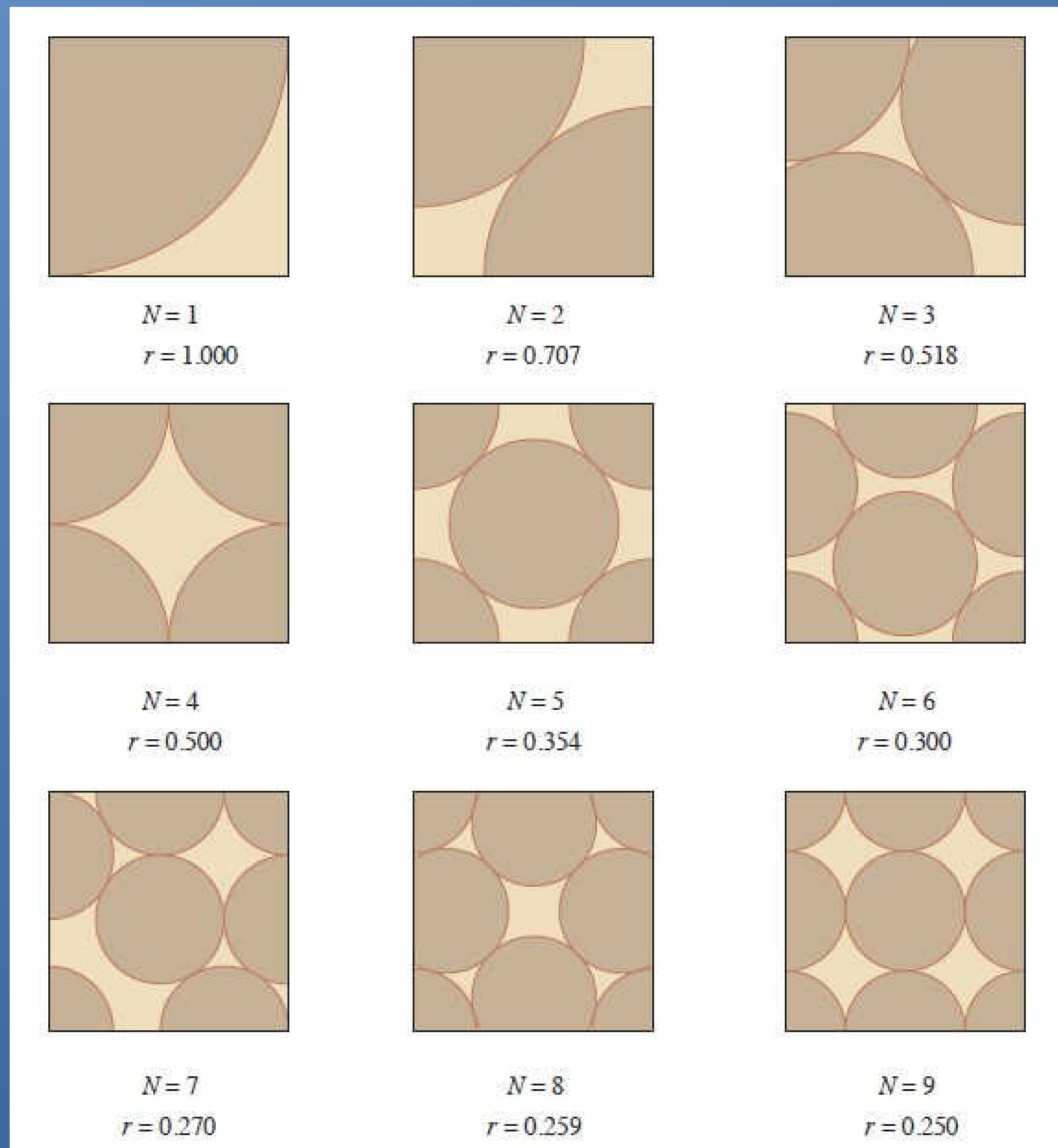
Origami Design Secrets, by Robert Lang

Circle Packing

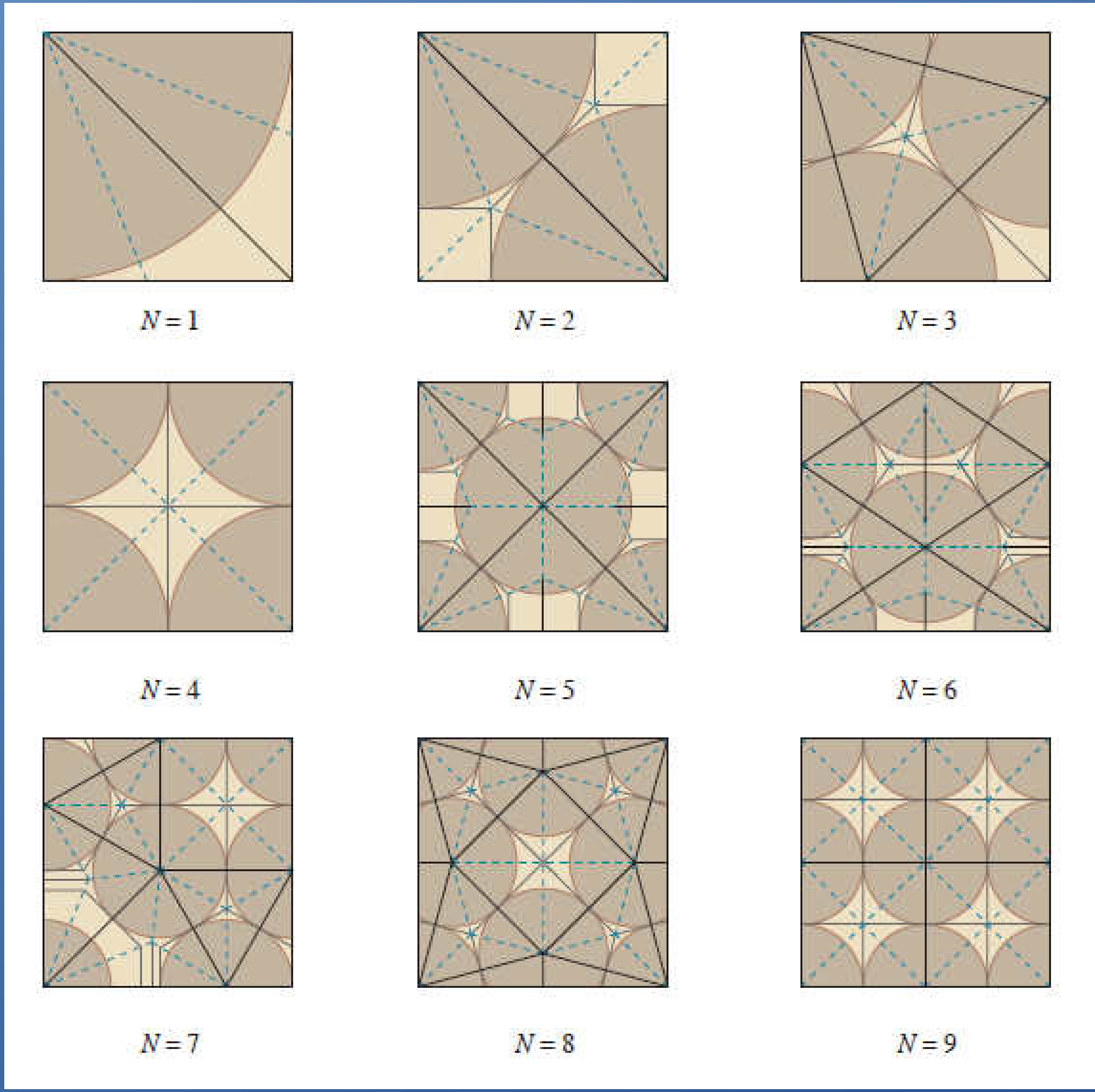




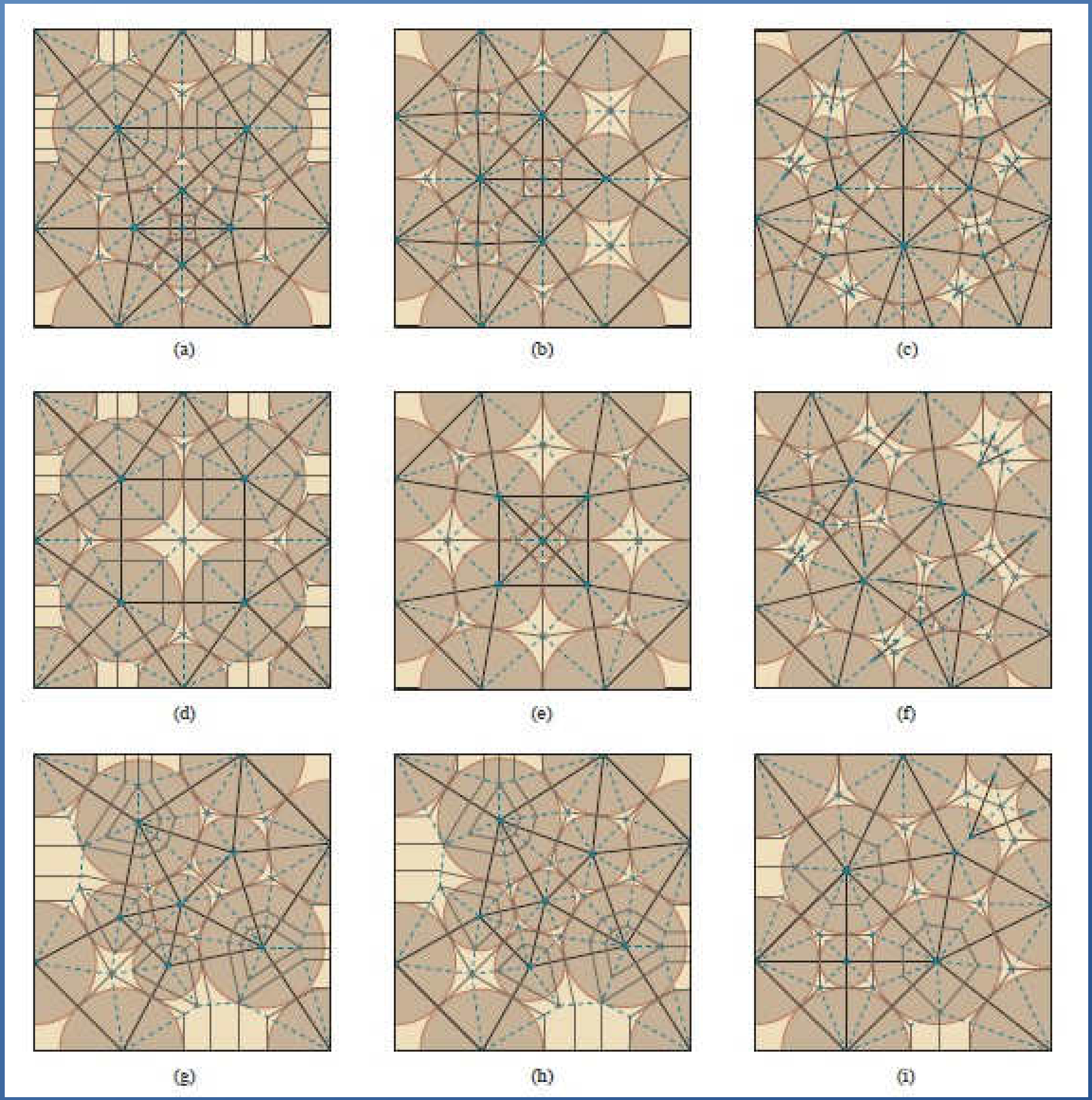
Circle Packing



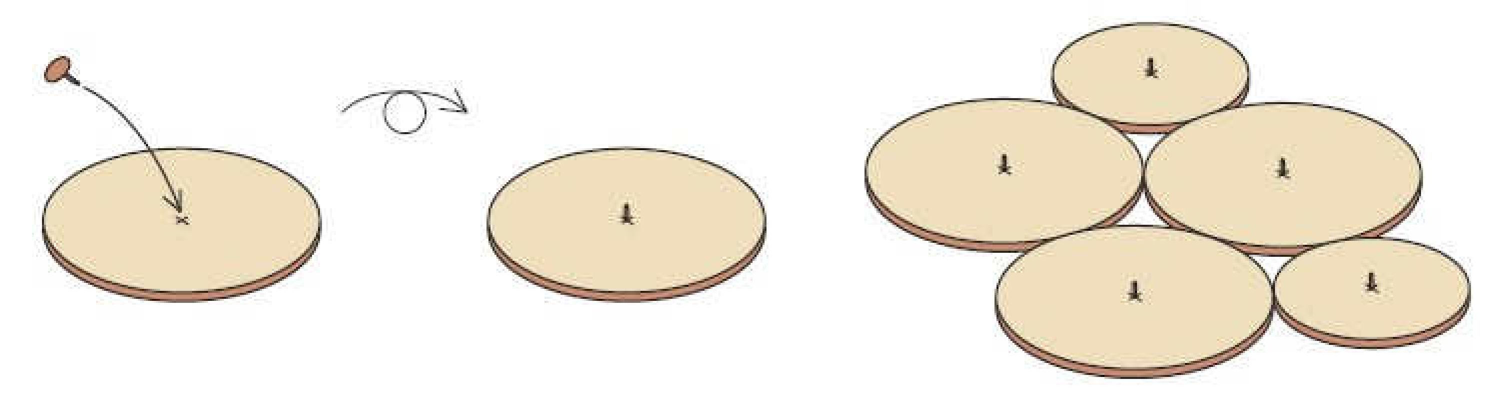
Circle Packing

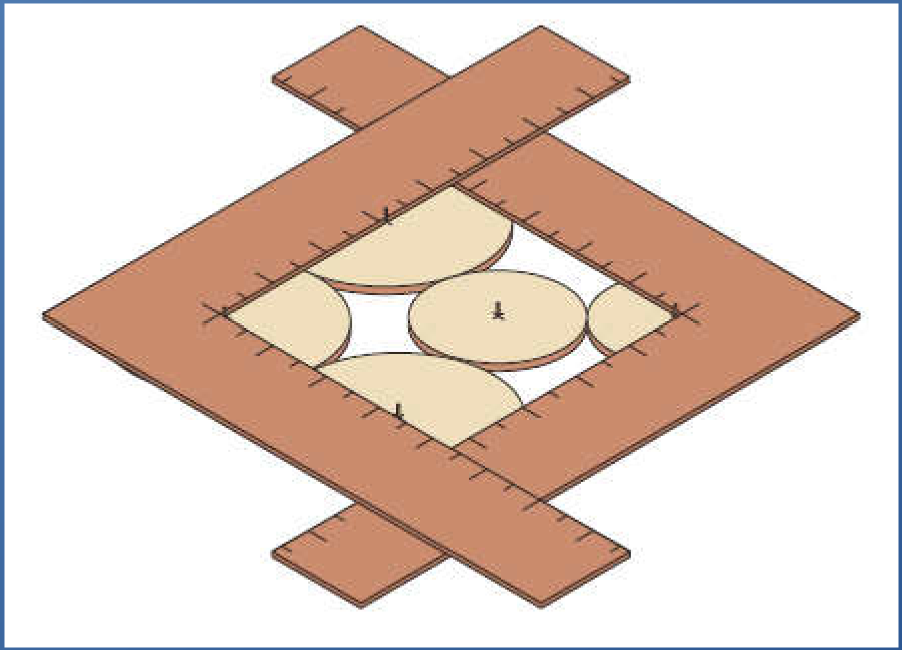


Circle Packing



Circle Packing



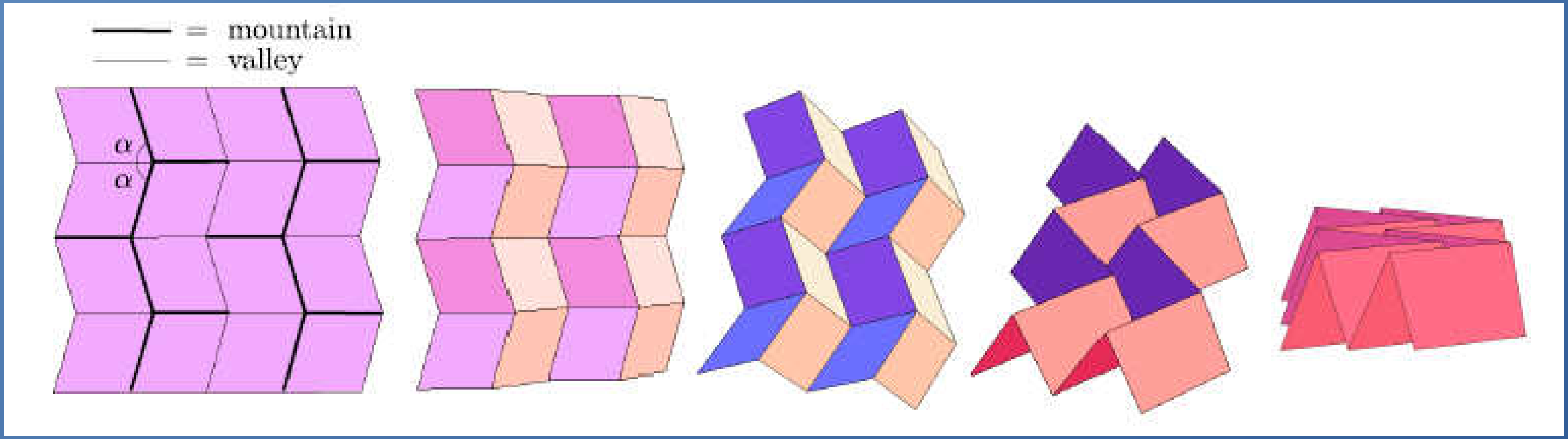


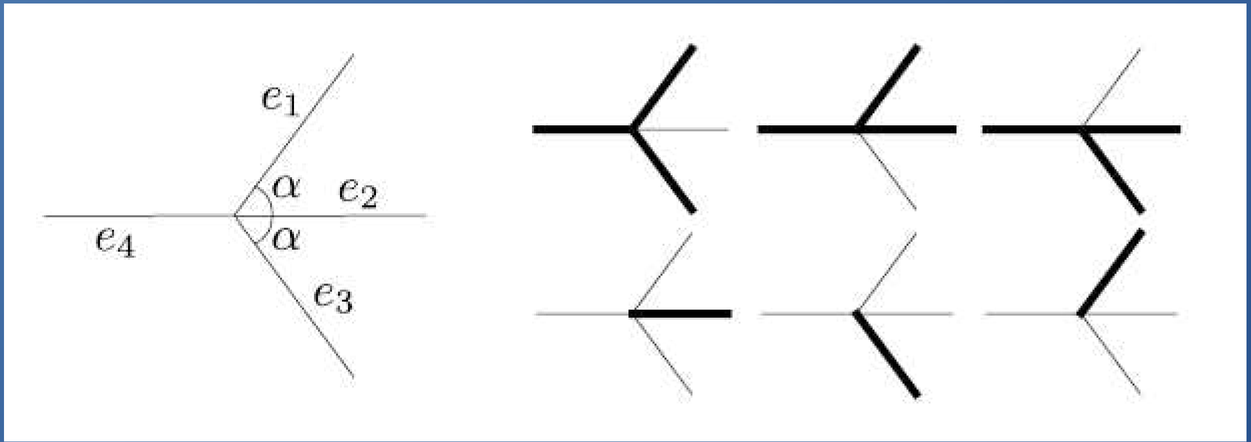
Circle Packing

- Software TreeMaker automates solving the circle packing problem
- Non-linear constrained optimization problem

Coloring Problems

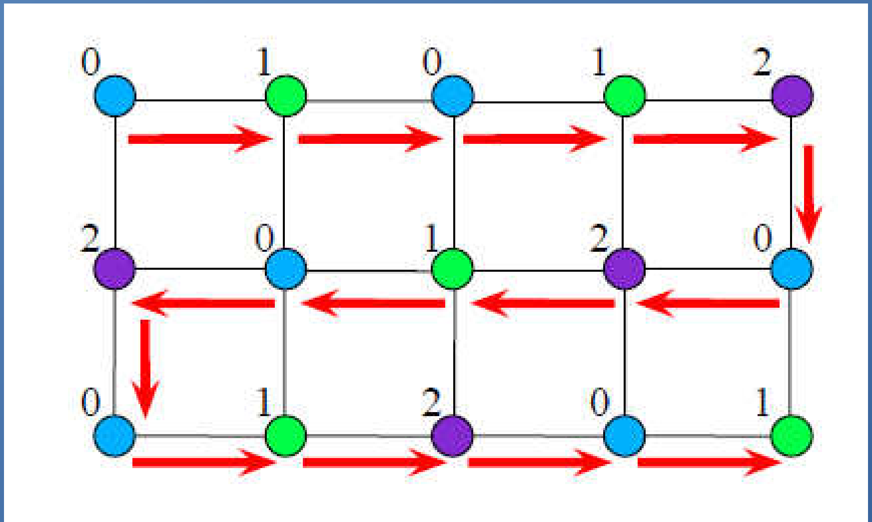
- Miura-ori: row staggered pattern
- One angle parameter

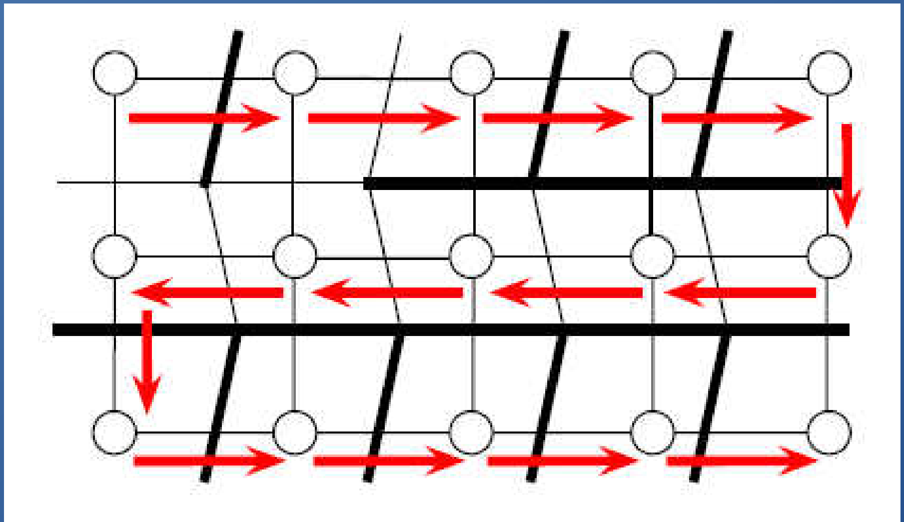




Coloring Problems

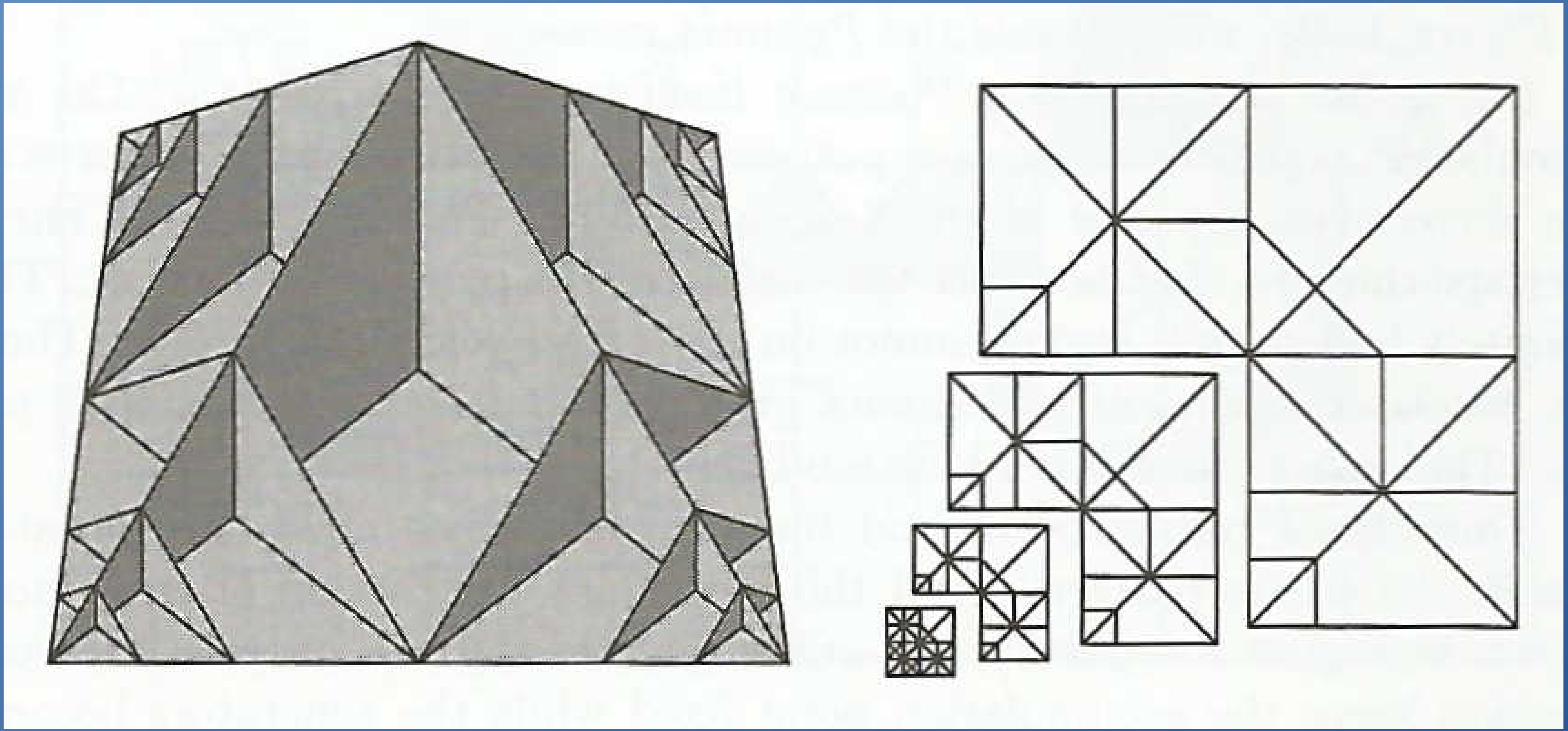
- Miura-ori: 3-colorings of the square lattice
- Equivalent to an ice problem in statistical mechanics
- Asymptotic number of colorings is $(4/3)^{3N/2}$



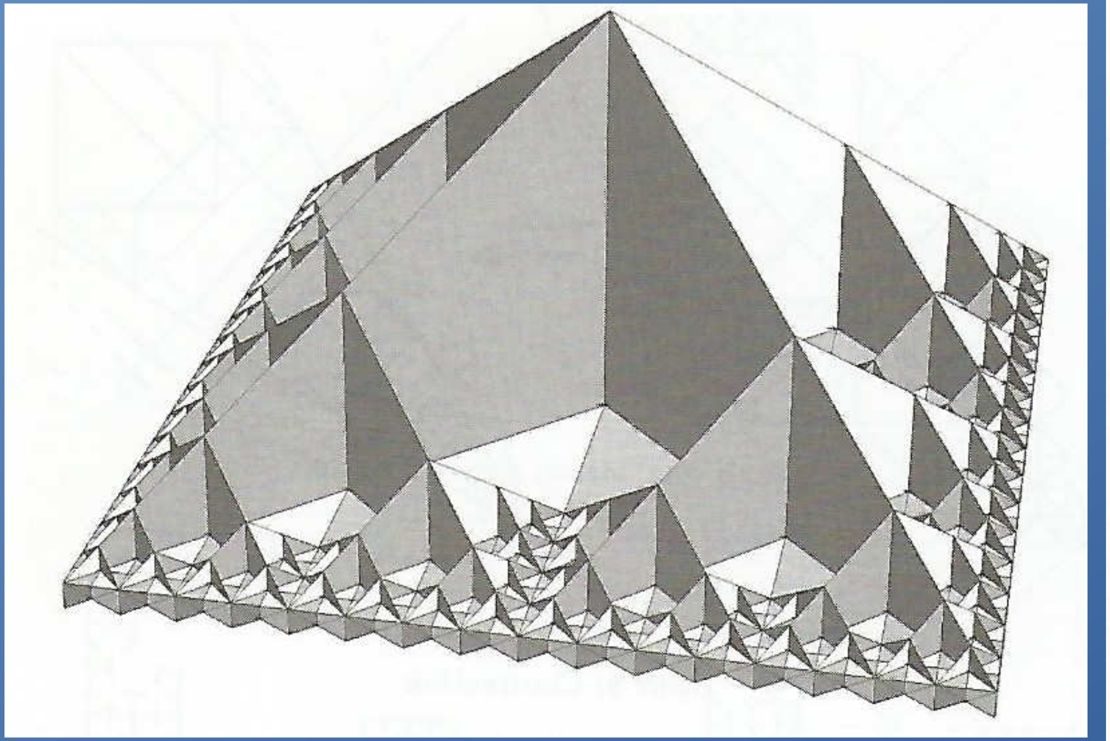


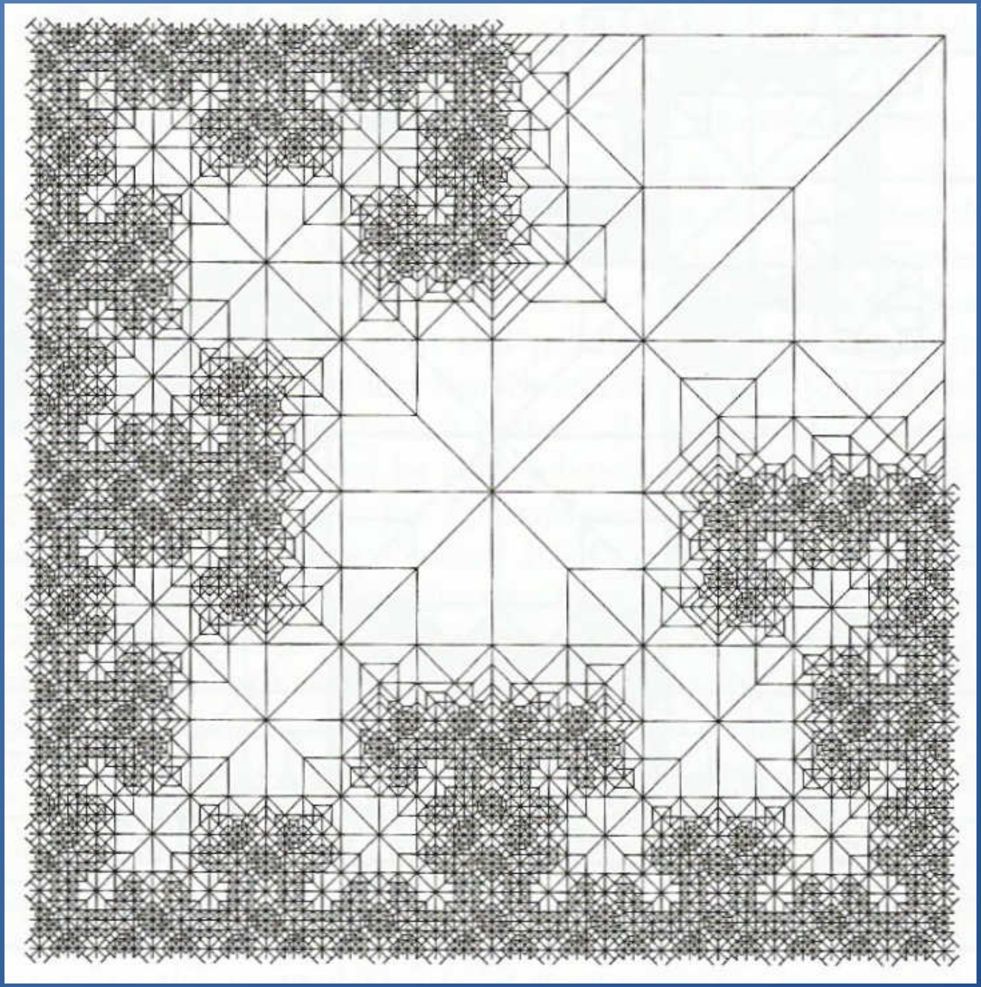
Beyond Flat 2D origami

Fractal Origami

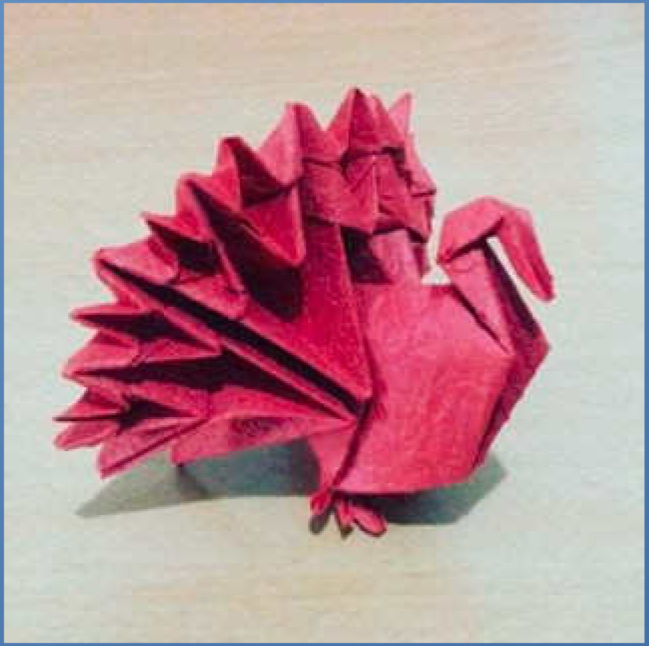


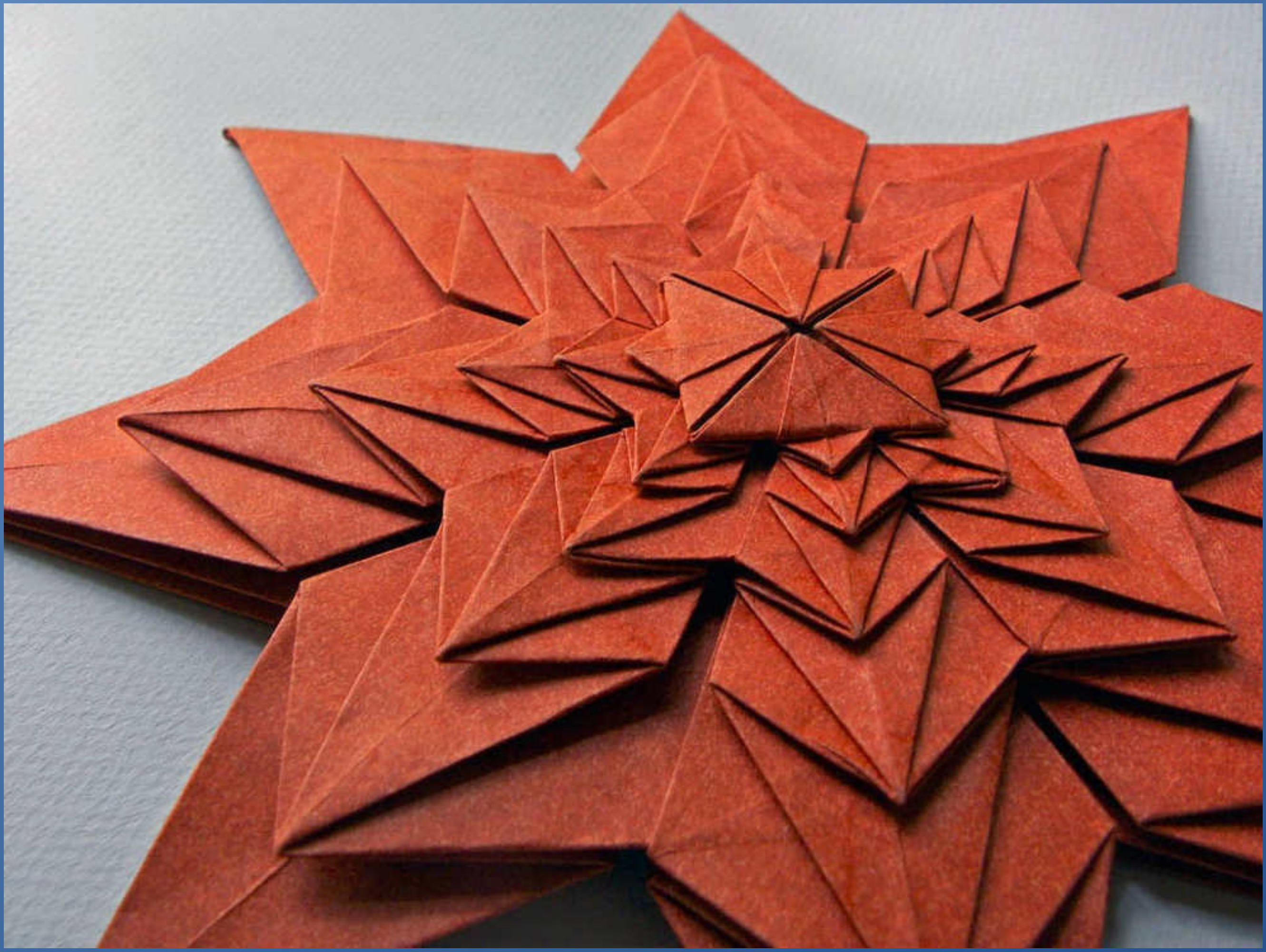
Fractal Origami





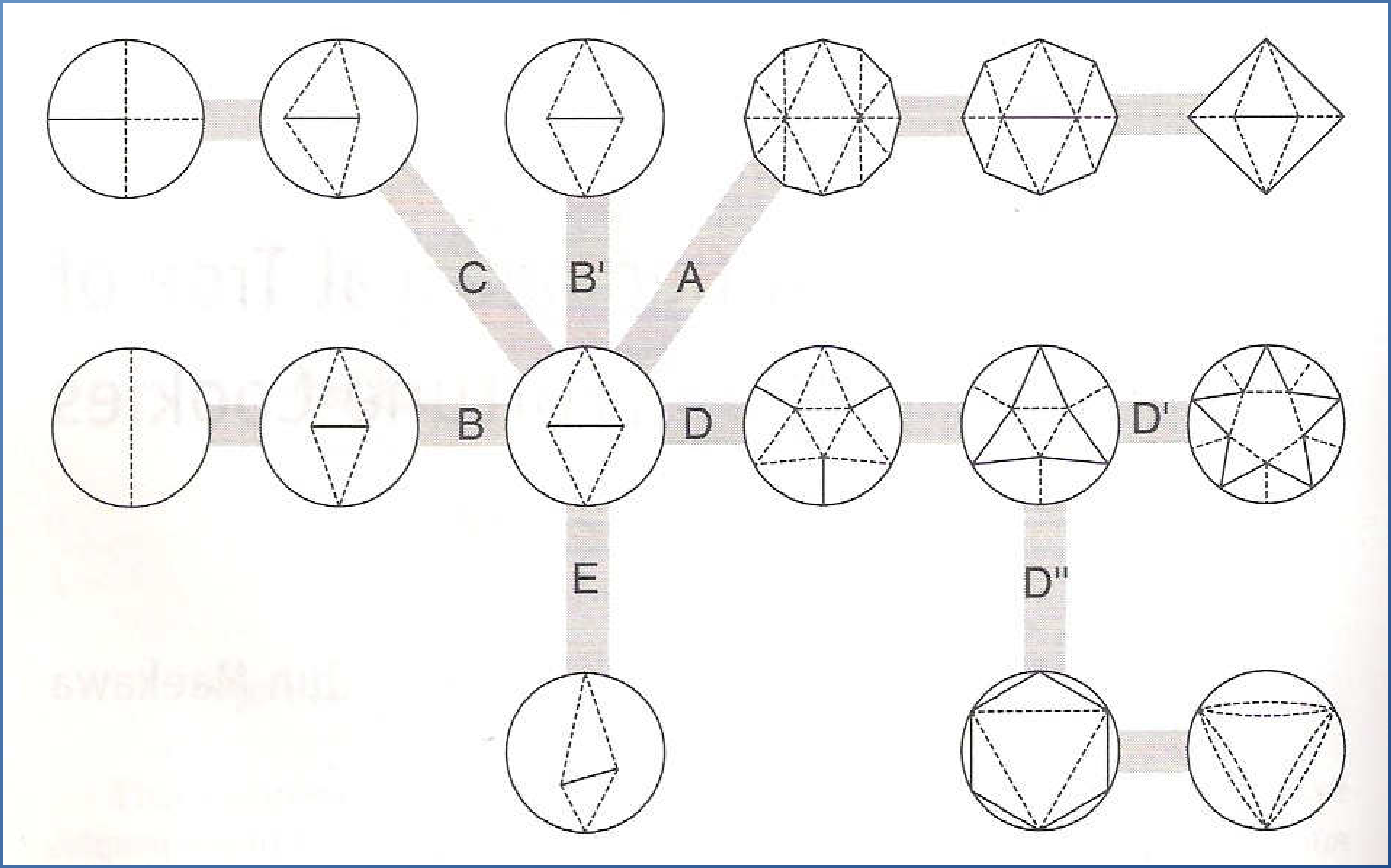
Fractal Origami



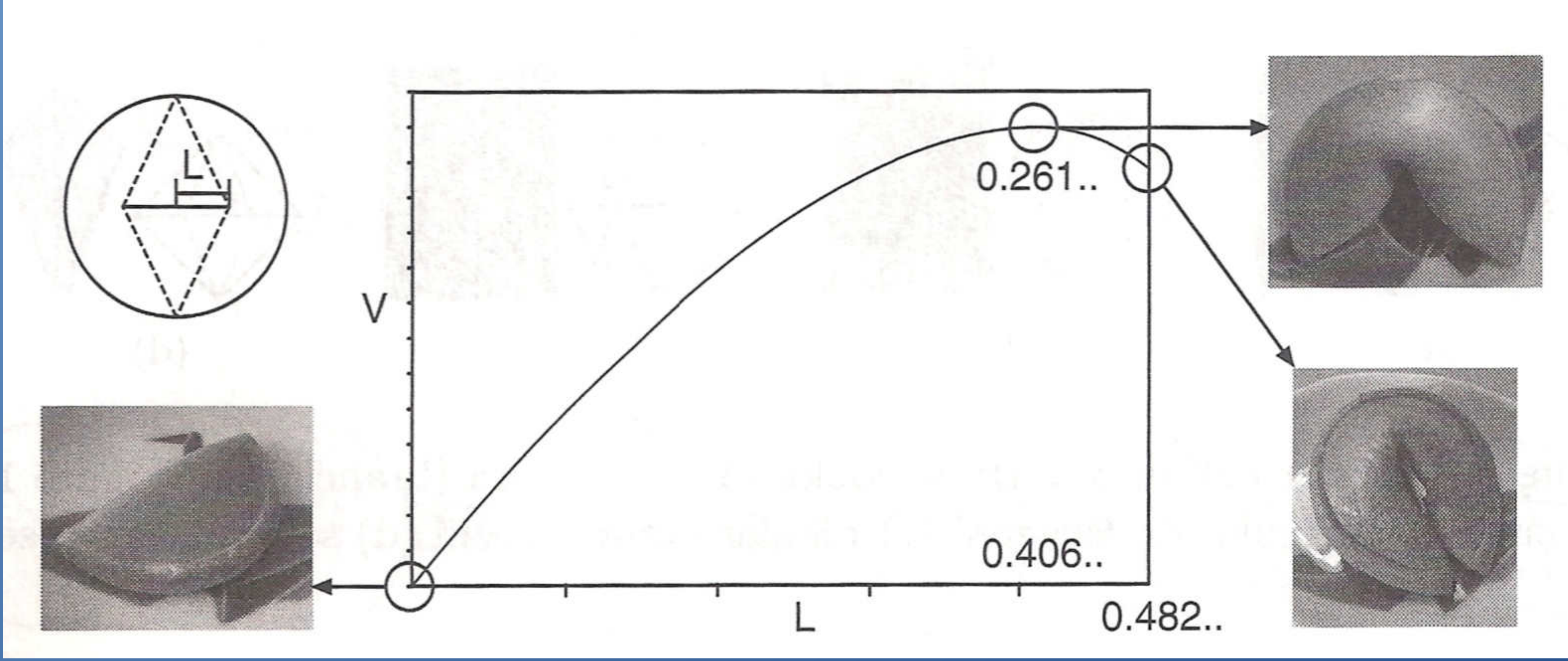


Breaking Flat Foldability

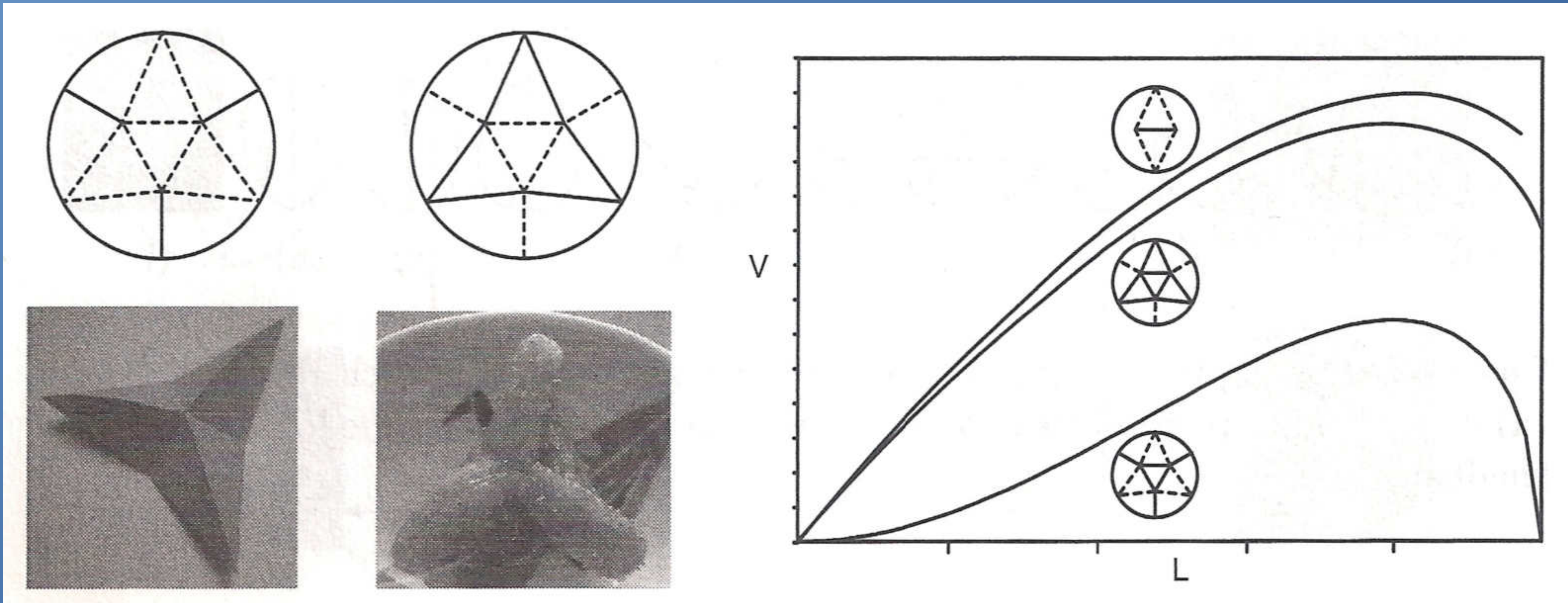
Breaking Flat Foldability



Breaking Flat Foldability



Breaking Flat Foldability



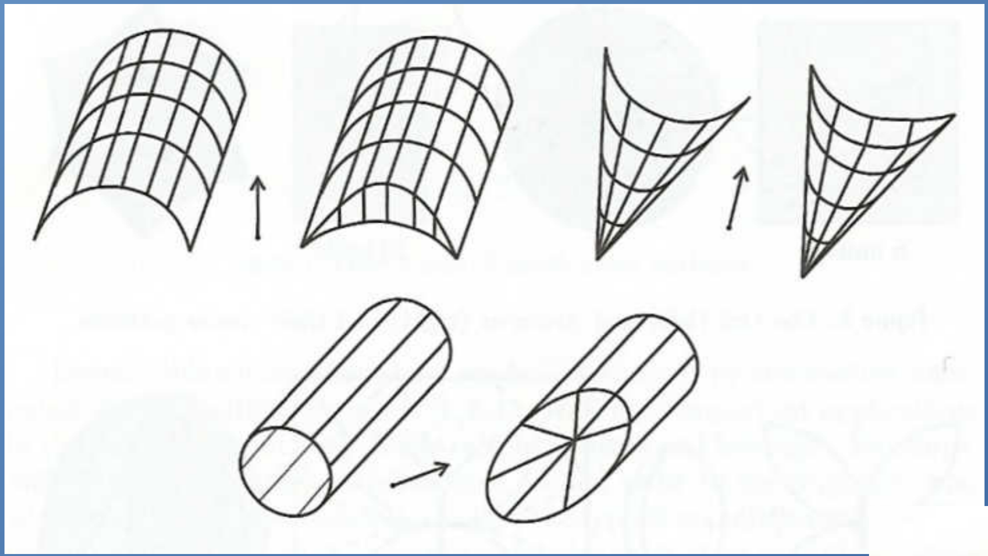
Breaking Flat Foldability

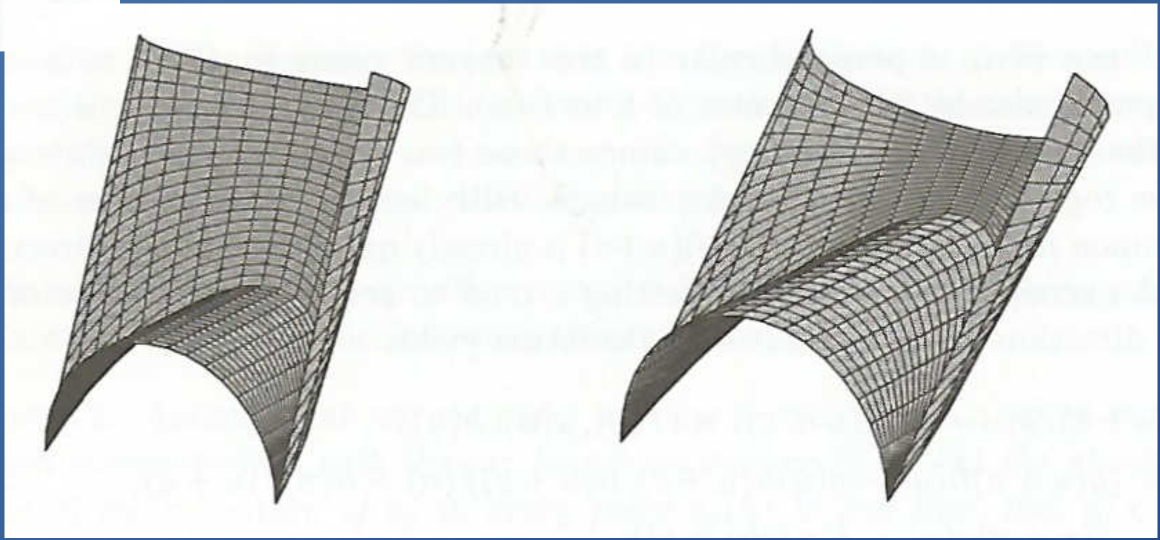
- Explorations:
  - Surface Area
  - Volume
  - Optimization problem
  - Other shapes

Non-flat paper

Non-flat paper

- Conics

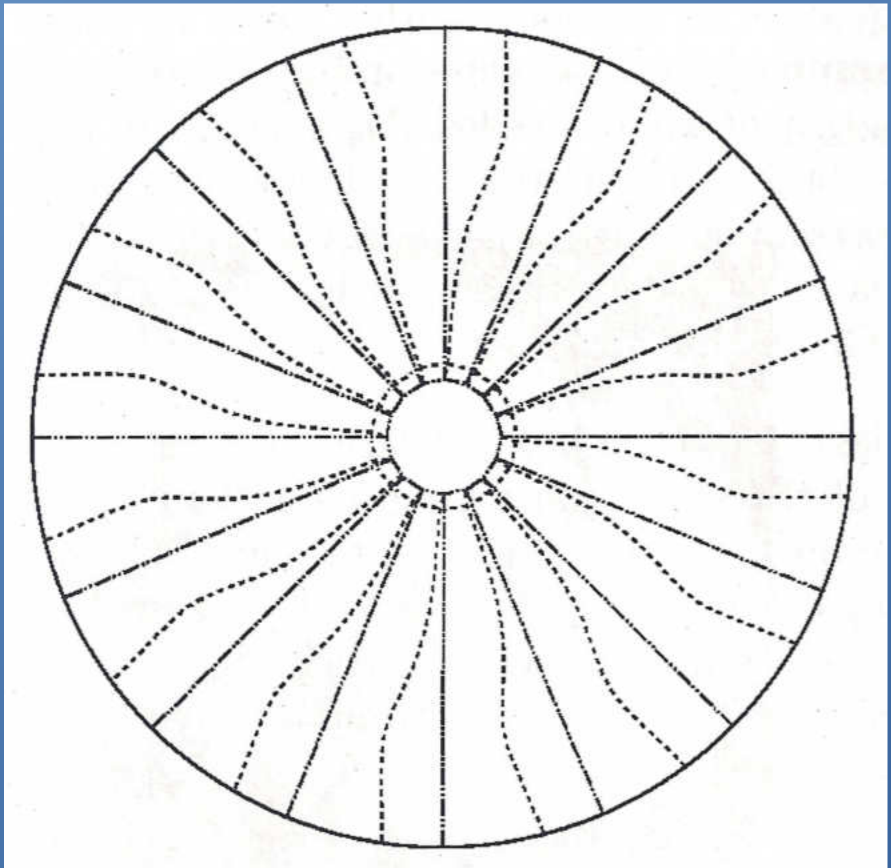


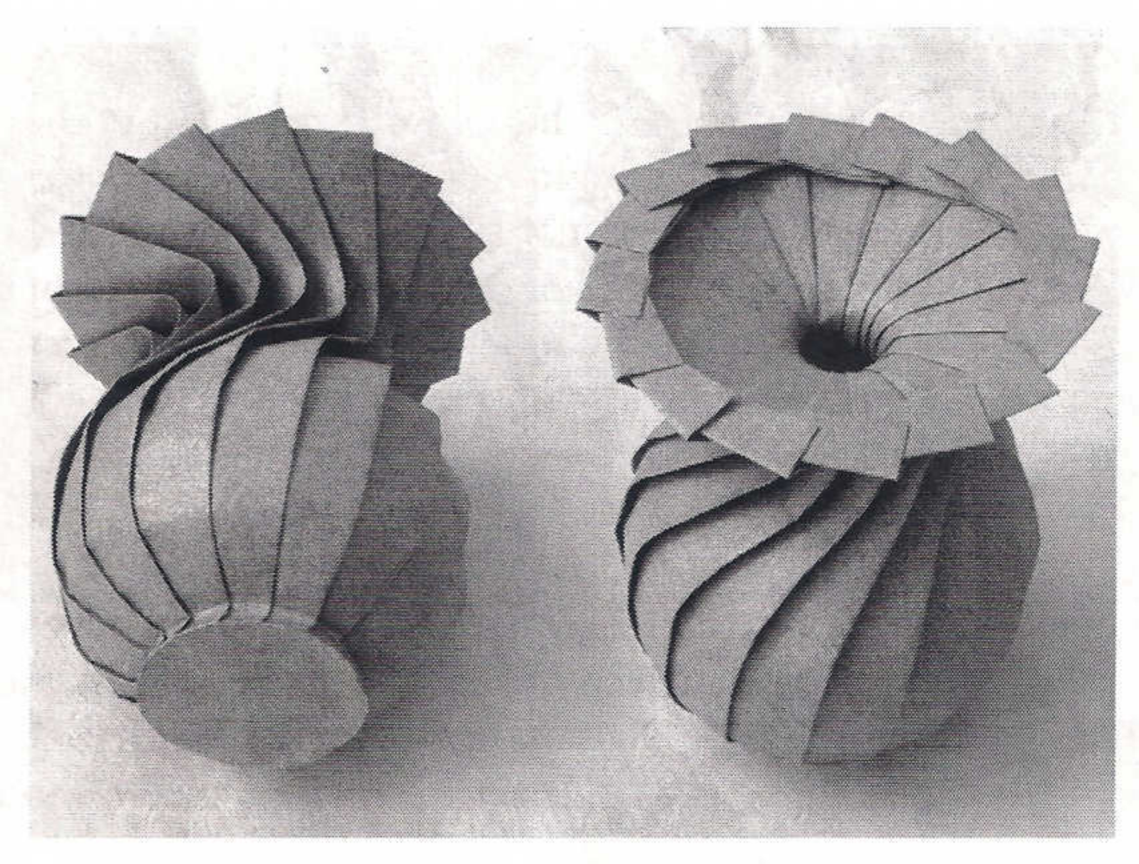


Non-flat paper

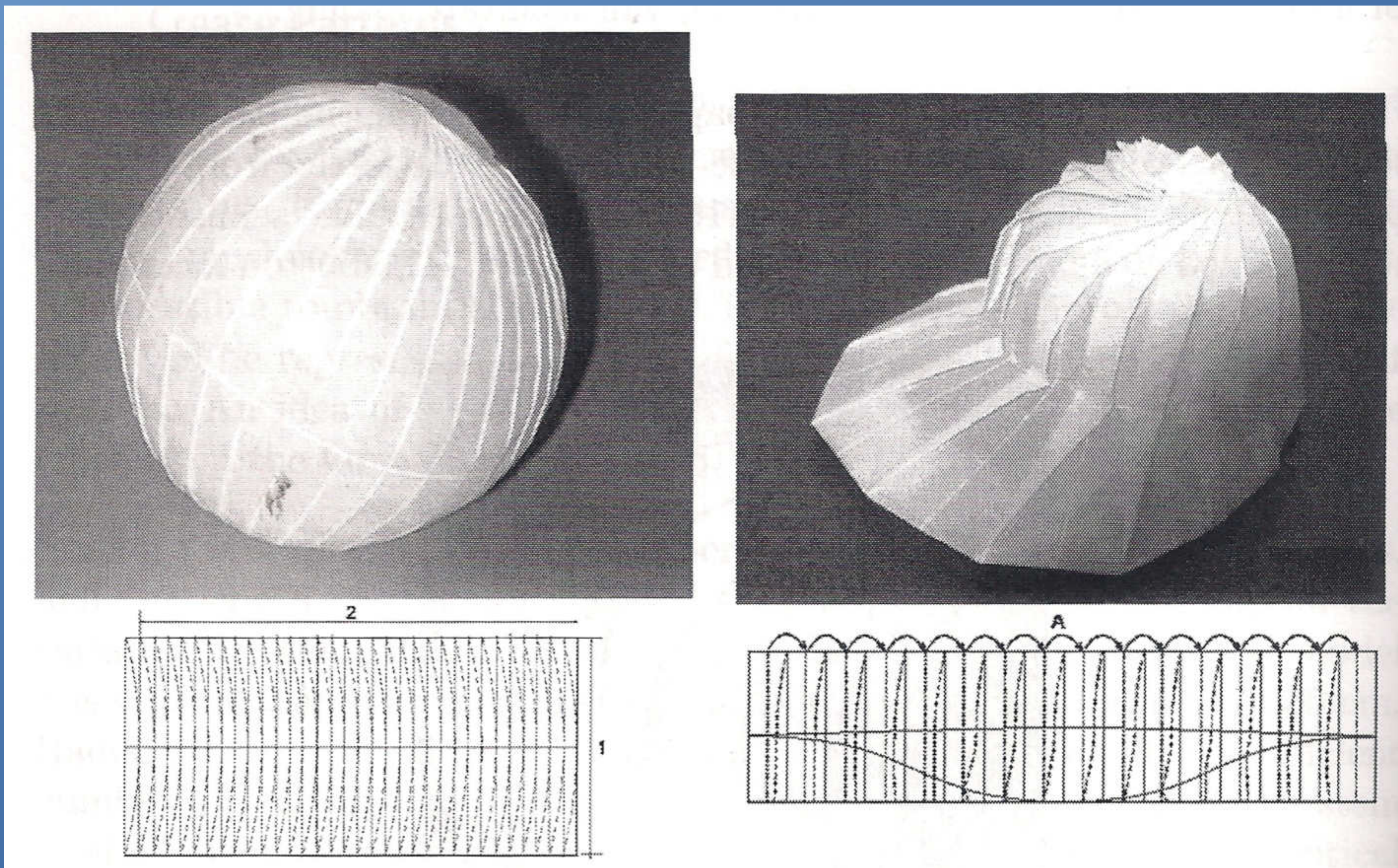
- Spherical paper, hyperbolic paper
  - One fold constructions are known

Curved Folding

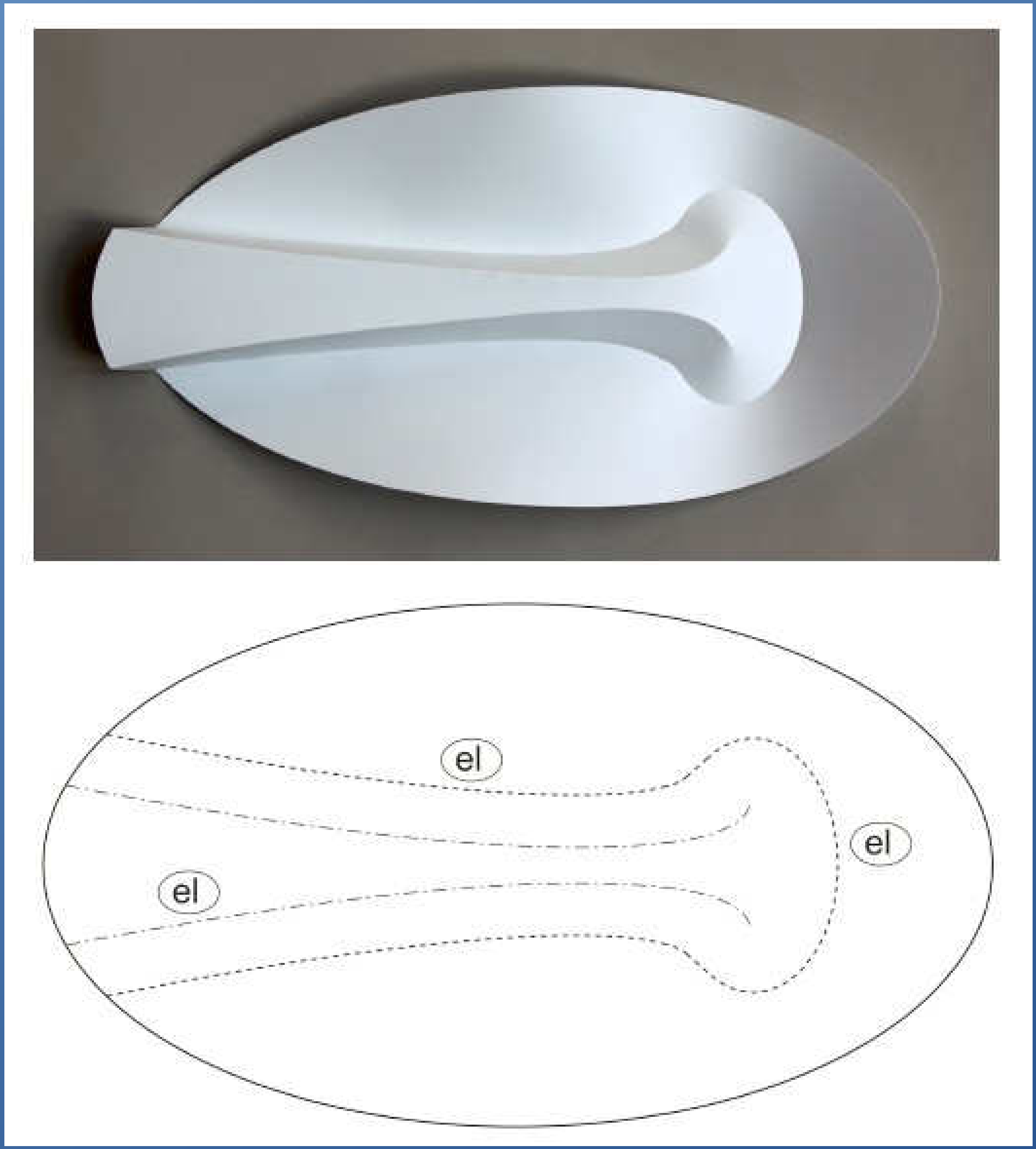




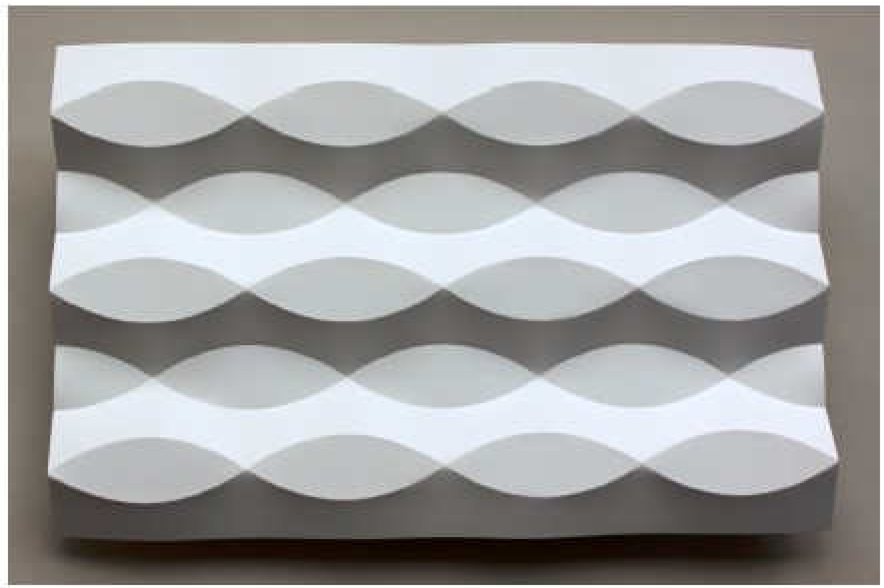
Curved Folding

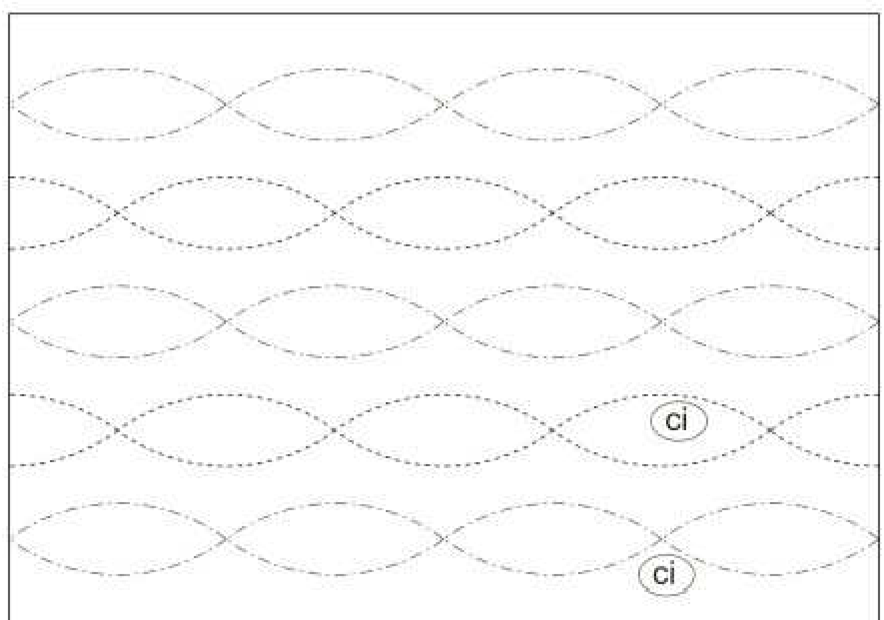


Curved Folding



Curved Folding





Curved Folding

Curved Folding

- No systematic algorithm for design known
- Direct applications in differential geometry
- Curved folding on non-flat paper not yet explored

A World Of Origami Maths

- Areas of mathematics involved only limited by imagination
- Many more applications in textbooks and convention proceedings
- Many simple research projects are awaiting students and teachers

Thank You!